The 29th International Conference on Difference Equations and Applications (ICDEA 2024)

International Society of Difference Equations (ISDE)

24 - 28 June 2024

Welcome Address

Letter of Organizing Committee

Dear participants,

Welcome to the 29th International Conference on Difference Equations and Applications (ICDEA) scheduled to take place from June 24 to June 28, 2024, in Paris (France). The conference will be held at the Holiday Inn Express Paris - Canal de la Villette in the city.

ICDEA 2024 is the annual conference of the International Society of Difference Equations (ISDE), a prestigious event that brings together researchers and scientists from around the world. The conference aims to foster the presentation, discussion, and exploration of solutions in the fields of Difference Equations, Discrete Dynamical Systems, and their applications to various disciplines such as mathematical biology, epidemiology, evolutionary game theory, economics, control theory, physics, and engineering.

Furthermore, we are pleased to announce that the conference this year is hosting 149 talks (106 in-person participants and 43 online participants). The submitted abstracts will be presented in Regular Sessions and Special Sessions. In addition, there will be 14 Plenary talks by leading experts in both the pure and applied aspects of difference equations and discrete dynamical systems.

We hope that your participation in this conference is both successful and productive, sparking new ideas and advancements in your field.

Best regards,

Thomas Michelitsch
Chair of the Organizing Committee

A letter from the President of ISDE

Dear Colleagues,

I am delighted to present the book of abstracts for the 29th International Conference on Difference Equations and Applications (ICDEA 2024). On behalf of the International Society of Difference Equations (ISDE), I extend my warmest greetings to all the distinguished scholars contributing to this conference.

This book showcases the depth and variety of scientific content in the talks at ICDEA 2024. The abstracts highlight diverse studies, analyses, and results from researchers exploring difference equations, discrete dynamical systems, and their applications in fields such as biology, economics, physics, and engineering. These abstracts illustrate the multifaceted nature of our discipline, covering fundamental aspects, mathematical properties, and novel methodologies.

I extend my deepest gratitude to the contributors whose work enriches this conference by advancing the theory of difference equations and discrete dynamical systems. The collected abstracts inspire further exploration in these fields. I also sincerely thank the organizing committee members for their tireless efforts in ensuring the conference's success and compiling this book.

As you read through this book, you will feel the spirit of collaboration, innovation, and intellectual curiosity that underpins this conference. Let these abstracts serve as catalysts for new ideas, fruitful discussions, and interdisciplinary collaborations. May they inspire researchers, educators, and practitioners to explore the fascinating world of difference equations and discrete dynamical systems.

Once again, I extend my warmest appreciation to all contributors. Your work and dedication are vital in shaping the future of our society, and it is an honor to have you as part of our esteemed community.

With sincere gratitude and best wishes,

Professor Laura Gardini
ISDE President

Conference Committees

Organizing Committee

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About ICDEA History

The previous conferences on Difference Equations and Applications were held in:

- 28th, 17-21 July 2023, Phitsanulok, Thailand
- 27th, 18-22 July 2022, Paris-Saclay, France
- 26th, Sarajevo, Bosnia and Herzegovina, July 26-30, 2021
- 25th, London, UK, June 24-28, 2019
- 24th, Dresden, Germany, May 21-25, 2018
- 23rd, Timisoara, Romania, July 24-28, 2017
- 22nd, Osaka, Japan, July 24-29, 2016
- 21st, Bialystok, Poland, July 19-25, 2015
- 20th, Wuhan, Hubei, China, July 21-25, 2014
- 19th, Muscat, Oman, May 26-30, 2013
- 18th, Barcelona, Spain, July 24-29, 2012
- 17th, Trois-Rivières, Quebec, Canada, July 24-29, 2011
- 16th, Riga, Latvia, July 19-23, 2010
- 15th, Estoril, Portugal, October 19-23, 2009
- 14th, Istanbul, Turkey, July 21-25, 2008
- 12th, Lisbon, Portugal, July 23-27, 2007
- 11th, Kyoto, Japan, July 24-28, 2006
- 10th, Münich, Germany, July 25-30, 2005
- 9th, Los Angeles, California, USA, August 2-6, 2004
- 8th, Brno, Czech Republic, July 28 - August 1, 2003
- 7th, Changsha, China, August 12-17, 2002
- 6th, Augsburg, Germany, July 20 - August 3, 2001
- 5th, Temuco, Chile, January 2-7, 2000
- 4th, Poznan, Poland, August 27-31, 1998
- 3rd, Taipei, Taiwan, 1997
- 2nd, Veszprém, Hungary, 1995
- 1st, San Antonio, Texas, USA, 1994

Contents

Plenary Speakers

Chaotic Attractors and their Bifurcations in 1D and 2D maps

Viktor Avrutin

Institute for Systems Theory and Automatic Control, University of Stuttgart, Germany

Abstract

Chaotic attractors can be considered from several different perspectives. They may be appreciated for their aesthetic beauty, or analyzed for the geometric properties, distinguishing between attractors with fractal and non-fractal dimensions. Keeping all these aspects in mind, in this talk, we consider chaotic attractors from the bifurcation theory point of view. Here, the central questions are how chaotic attractors appear and which transformations they may eventually undergo. There is a long tradition behind the first question, frequently referred to as "routes to chaos". As for the second question, the complete picture is still missing. It is known that many of such transformations are related to some homoclinic bifurcations, leading, for example, to a sudden expansion of a chaotic attractor (so-called interior crises, also known as expansion bifurcations).

In addition to that, in piecewise smooth systems, robust chaotic attractors may undergo border collision bifurcations. In general, these transformations of chaotic attractors are not related to homoclinic bifurcations and can be explained by mainly geometric reasoning. The effects caused by border collision bifurcations of chaotic attractors are diverse. In particular, such a bifurcation may cause an arbitrary number of additional bands of the attractor to occur, as well as an arbitrary number of additional holes inside the existing bands. Under specific conditions, more effects

are possible, including a sudden expansion of the attractor or a sudden jump in its fractal dimension (a transition from a non-fractal to a fractal chaotic attractor).

This presentation will provide an overview of bifurcation leading to transformations of chaotic attractors, starting with classical scenarios related to homoclinic bifurcations and concluding with current findings on border collision bifurcations of chaotic attractors.

References:

[1] V. Avrutin, A. Panchuk and I. Sushko, Border collision bifurcations of chaotic attractors in one-dimensional maps with multiple discontinuities, Proc. R. Soc. A, 477, 20210432, 2021.

[2] V. Avrutin, A. Panchuk and I. Sushko, Can a border collision bifurcation of a chaotic attractor lead to its expansion?, Proc. R. Soc. A, 479, 20230260, 2023.

[3] V. Avrutin and I. Sushko, Border collision bifurcations of chaotic attractors in two-dimensional discontinuous maps, forthcomnig.

Biography

Viktor Avrutin is a professor at the University of Stuttgart, Germany. His primary research focus is on piecewise smooth dynamical systems in discrete time, a field he began exploring during his PhD and Habilitation thesis. Since then, on this subject he has published (with some collaborators) nearly one hundred papers in International journals, and several other publications.

Worth mentioning are also his Habilitation thesis, which delves into complex bifurcation structures within robust chaos, as well as the co-authored book Viktor Avrutin et al., Continuous and Discontinuous Piecewise-Smooth One-Dimensional Maps: Invariant Sets and Bifurcation Structures, World Scientific 2019.

His main research activities concentrate on low-dimensional discontinuous maps, smooth and piecewise smooth, continuous and discontinuous. In particular, he is interested in the bifurcations that PWS systems may exhibit and the bifurcation scenarios they may form. One more long-term research interest of Viktor Avrutin are chaotic attractors and bifurcations they may be involved in. This will be the topic of the talk.

Bifurcations of tori

Soumitro Banerjee

Indian Institute of Science Education & Research, Kolkata, India

Abstract

It is known that invariant closed curves in maps, representing tori in continuous-time phase space, undergo various bifurcations. This includes two types of torus doubling, the generation of a third frequency, and the merger and disappearance of stable and unstable invariant closed curves. In the past, a few approaches have been proposed to analyse and predict the outcome of such bifurcations. We review these approaches and show that some of these may fail under certain conditions. We propose an alternative approach that applies to both resonant and ergodic tori.

Biography

Dr. Soumitro Banerjee earned his B.E. in Electrical Engineering from the Bengal Engineering College in 1981, M.Tech. from IIT Delhi in 1983, and Ph.D. from the same Institute in 1988. He was in the faculty of the Indian Institute of Technology, Kharagpur, for 23 years and moved to the Indian Institute of Science Education & Research, Kolkata, in 2009. He has published four books: ”Nonlinear Phenomena in Power Electronics” (IEEE Press, 2001), ”Dynamics for Engineers” (Wiley, London, 2005), ”Wind Electrical Systems” (Oxford University Press, New Delhi, 2005), and ”Research Methodology for Natural Sciences” (IISc Press, 2022). He received the S. S. Bhatnagar Prize (2003) and was recognized as a ”Highly Cited Author” by Thomson Reuters from 2004 to 2014. He is a Fellow of the Indian Academy of Sciences, the Indian National Academy of Engineering, the Indian National Science Academy, The World Academy of Sciences, and the IEEE.

The Beverton–Holt Equation

Martin Bohner

Missouri S&T, USA E-mail: bohner@mst.edu

This is joint work with Jaqueline Mesquita (University of Brasilia, Brazil) and Sabrina Streipert (University of Pittsburgh, USA)

Abstract

In this talk, we will present the Beverton–Holt equation as used in fisheries and other population models, in many different scenarios (discrete case, continuous case, time scales case, quantum case, periodic case, with and without harvesting etc.).

References:

[1] M. Bohner, J. Mesquita, and S. Streipert, Periodicity on isolated time scales. Math. Nachr., 295(2):259–280, 2022.

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[11] M. Bohner and H. Warth. The Beverton–Holt dynamic equation. Appl. Anal., 86(8):1007–1015, 2007.

Biography

Martin Bohner is the Curators' Distinguished Professor of Mathematics and Statistics at Missouri University of Science and Technology in Rolla, Missouri, USA. He received the BS (1989) and MS (1993) in Econo-mathematics and PhD (1995) from University Ulm, Germany, and MS (1992) in Applied Mathematics from San Diego State University. He was a Postdoc, sponsored by the Alexander von Humboldt-Foundation, at National University of Singapore (1997) and at San Diego State University (1998). Martin Bohner is a Past President of ISDE, the International Society of Difference Equations. His research interests center around differential, difference, and dynamic equations as well as their applications to economics, finance, biology, physics, and engineering. He is the author of seven textbooks and more than 350 publications, Editor-in-Chief of five international journals, and Associate Editor for almost 100 international journals. His work has been cited more than 20,000 times in the literature, including more than 5000 citations of his book "Dynamic Equations on Time Scales: An Introduction with Applications", co-authored with Professor Allan Peterson. His h-index is 64, and his i10-index is 243. Professor Bohner is the recipient of the 2021 Obada Prize. His honors at Missouri S&T include five Faculty Excellence Awards, one Faculty Research Award, and eight Teaching Awards.

From continuous delayed differential systems to discrete models: can stochastic perturbations improve control outcome for difference equations?

Elena Braverman (joint talk with Alexandra Rodkina)

University of Calgary

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Abstract

Differential and difference equations provide an adequate description of physical and biological phenomena. Significant interest to discrete models is stimulated by complicated types of behaviour exhibited even by simple maps. Differential equations have less sophisticated dynamics unless delay is included. Delay differential equations combine features of ordinary differential and difference equations, including chaotic oscillations in equations, not system. Chaos, though predicted in discrete maps, is not as easily observed in nature as cyclic dynamics, and stochastic perturbations are considered as a regulating force. Noise is an integral part of our world, it can be intrinsic, due to internal system processes, as well as extrinsic, due to the influence of the environment. To exclude undesired behaviour of discrete maps, control can be introduced, which can also involve a stochastic component.

Introduction of noise into discrete modeling, as well as into controls, is quite a natural part of a model design. While sometimes noise destroys stability of a system, we concentrate on the opposite situation, when deterministic control does not stabillize but stochastic one with the same expected value does. For difference equations and systems, we investigate the influence of stochastic perturbations on population survival, chaos control, eventual cyclic behavior, and on local and global stability. We distinguish between the cases when noise can and when it cannot improve stability. The main purpose is to highlight an active role of stochastic perturbations in stabilization of controlled difference models.

Biography

Elena Braverman got her PhD at the Urals State University, Ekaterinburg, Russia. After two postdoctoral positions at the Technion, Israel and Yale University, USA, she joined the University of Calgary in 2002, where she has been a professor since 2011. E. Braverman is a Fellow of the Canadian Mathematical Society (2023), and a co-author of about 200 scientific papers and a monograph.

Chaos in Finite-Dimensional Linear Systems with Weak Topology

Guanrong (Ron) Chen

City University of Hong Kong

Abstract

In this talk, we discuss a case of Li-Yorke chaos from a linear system of differential equations in a finite-dimensional Euclidean space with a weak topology, where a solution flow of the system is proved to be Li-Yorke chaotic under certain conditions. This is in sharp contradiction to the well-known fact that linear differential equations cannot be chaotic in a finite-dimensional space with a strong topology. We will also show that there is a sequential version of Li-Yorke chaos generated by iterating a bounded linear map on a finite-dimensional space with a weak topology under some conditions.

Biography

Professor Chen is the "Shun Hing Education and Charity Fund Chair Professor in Engineering" at City University of Hong Kong. From 1987 to 1990 he worked at Rice University and from 1990 to 1999 at the University of Houston. Since 2000, he works as a chair professor at City University of Hong Kong, founding the Centre for Complexity and Complex Networks. He is known for his contributions to chaos theory and bifurcation analysis, nonlinear dynamics, complex networked systems, constructing the Chen attractor, the Lu-Chen attractor and various multi-scroll attractors. He has published over 700 journal papers and some 300 conference abstracts on chaos theory, complex dynamical networks and closely related fields. He currently received over 100,000 citations, making him one of the most cited researchers in his fields. He was elected IEEE Fellow in 1997 and is now Life Fellow. He was awarded the 2011 Euler Gold Medal from Russia and conferred Honorary Doctor Degrees by the Saint Petersburg State University, Russia in 2011 and by the University of Le Havre, France in 2014. He is a Member of Academia Europaea and a Fellow of The World Academy of Sciences. In August 2024 he will receive the C. S. Hsu Award for Distinguished Scholars in Nonlinear Dynamics and Control, issued annually by the Nonlinear Science and Complexity Conferences Series. From 2009 to 2014, he served as the chairman for the Complex Systems and Networks Committee of the Chinese SIAM Society. Since 2010, he has been serving as the Editor-in-Chief for the International Journal of Bifurcation and Chaos.

Discrete-time Darwinian Dynamics

Jim Cushing

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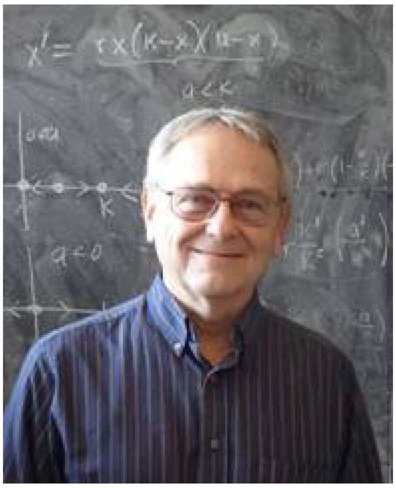

Abstract

Difference equations have a long history of use in defining discrete-time models for the dynamics of biological populations. The vast majority of difference equations that have been used for this purpose are time autonomous and, therefore, assume that model coefficients remain forever constant in time. This assumption is, of course, biologically unrealistic because the coefficients represent biological parameters and mechanisms that, for any number of reasons, typically change over time, especially in natural settings (but also in even controlled settings). One particularly notable reason for this is related to the most basic principle in biology, namely Darwinian evolution by natural selection. While evolution is often viewed as a slow process (in generation time units), it has been observed in recent decades that it can proceed at a remarkably fast pace (even within a few generations), in both natural and laboratory circumstances. This emphasizes that it is of interest to include evolution population models, especially those whose asymptotic dynamics are investigated.

In this lecture I will review one methodology for including evolution in any discrete-time population model of interest, namely the methodology of Darwinian dynamics (sometimes called evolutionary game theory). I will describe a few general theorems about the equilibrium dynamics of such Darwinian models (motivated by the extinction versus survival struggle). I will briefly discuss trait (strategy) driven-evolution and invasion-driven evolution with the associated concept of an evolutionarily stable strategy (ESS). Darwinian versions of the classic discrete logistic (Beverton-Holt) equation will be used as simple illustrative examples.

Biography

Jim Cushing holds the position of Professor Emeritus at the University of Arizona, Tucson, USA, with affiliations with the Department of Mathematics and the Interdisciplinary Program in Applied Mathematics. He earned his doctorate in applied mathematics in 1968 at the University of Maryland in College Park, USA, in affiliation with the Institute of Fluid Dynamics. After studying the partial differential equation models of deep water waves for several years, his interests turned to population and ecological dynamics, on which he focused for over 45 years. He has collaborated on interdisciplinary projects with laboratory experimentalists, field ecologists and, pedagogically, with members of the Ecology and Evolutionary Biology Department at the University of Arizona. In these collaborations he has utilized difference, ordinary and partial differential, integral, and integro-differential equations with a concentration on asymptotic dynamics. He is the author of over 180 research articles and the author/coauthor of six books.

Spatial discretization of dynamical systems

Peter Kloeden

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Abstract

Many insights into complex behaviour of nonlinear dynamical systems are derived from computer studies, so it is important to understand just what effects arise from simulation of continuous processes on the discrete phase space of a finite state machine. This is not the same as the accuracy of representing differential equations by finite difference schemes, but concerns the consequences of discretizing the state space for system dynamics whose "true" values lie in a continuum.

Such spatial discretizations can produce artifacts such as spurious fixed points and limit cycles that do not exist in the true system evolution in a continuous space. A basic problem here is that any trajectory of the discretized system is eventually periodic, even if. the original system is chaoztic.

What then is the relationship between a chaotic dynamical system and the behaviour of systems based on spatial discretizations?

It will be shown here that the appropriate characterizations to investigate and compare are invariant measures. There are two essentially equivalent approaches depending on how the discretized function is constructed: a random choice of closest values on the discretized space or a set-valued choice of all such points.

References:

[1] P. Diamond and P. E. Kloeden, Spatial discretization of mappings, *J. Computers Math. Applns.* **26** (1993), 85-94.

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[4] P. Imkeller and P.E. Kloeden, On the computation of invariant measures in random dynamical systems, *Stochastics & Dynamics* **3** (2003), 247–265.

Biography

Prof. Dr. Peter E. Kloeden has wide interests in the applications of mathematical analysis, numerical analysis, stochastic analysis, and dynamical systems. He is the coauthor of several influential books on non-autonomous dynamical systems, metric spaces of fuzzy sets, lattice systems, and stochastic numerics. Prof. Kloeden is a Fellow of the Society of Industrial and Applied Mathematics. He was awarded the W.T. & Idalia Reid Prize from SIAM in 2006. His current interests focus on non-autonomous and random dynamical systems and their applications in the biological sciences. He retired as Professor of Applied and Instrumental Mathematics at the Goethe University in Frankfurt am Main in 2014 and was then a "thousand expert" professor at the Huazhong University of Science and Technology in Wuhan. He is currently a guest researcher at the University of Tuebingen.

Consensus in Systems Defined on Time Scales and Fractional-Order Systems

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Abstract

Opinion dynamics is a field of study that encompasses mathematical models describing interactions among groups of entities known as agents, including humans, schools of fish, or swarms of robots. This talk focuses on a fundamental phenomenon in such systems, namely, the emergence of agreement or consensus. Consensus occurs when a group of agents agree on shared values or opinions, such as position, velocity, or price. By incorporating fractional difference operators instead of classical ones, memory effects are introduced into the system dynamics. It's worth noting that individuals, animals, and electronic systems possess memory, which influences their behaviors. Models of opinion formation are particularly relevant when analyzed on discrete-time domains, where agent meetings and information exchanges occur intermittently. Leveraging time scales theory allows for a flexible exploration of various sampling or time step configurations in these processes. Results obtained highlight the significance of both memory and the duration of time gaps between information exchanges among agents in shaping opinion dynamics and achieving consensus.

In situations where consensus is not reached, polarization or chaos may ensue. In such cases, one strategy to steer all agents toward consensus involves introducing a (virtual) leader and implementing control mechanisms within the system. This concept draws parallels from real-world phenomena, such as the relationship between a sheepdog and sheep or the influence of mass media on societal opinions. Controlling the system through a leader finds practical applications, such as crowd evacuation during emergencies or designing reference trajectories for guiding groups of robots. Control strategies may vary, depending on factors such as the coupling strength between agents or the desired level of external intervention. These strategies will also be presented.

Through rigorous research and analysis, this talk offers insights into control strategies tailored for achieving consensus in fractional-order systems and systems operating on time scales.

Biography

Agnieszka B. Malinowska is a PhD, DSc, Associate Professor, and the Director of the Institute of Information and Communication Technology at the Faculty of Computer Science, Bialystok University of Technology. She has authored or coauthored 57 papers in international journals with Impact Factor (IF), 14 book chapters, 22 papers in journals without IF, 4 monographs, and 22 conference papers. Her work has received 1389 citations (excluding self-citations) on Web of Science (h-index = 22) and 2259 citations on Scopus (h-index = 25). Her research interests include fractional calculus and its applications, as well as time scale theory and its applications. She has participated in various research projects, including those funded by the Polish National Science Center and international collaborations such as the NECTAR project. Dr. Malinowska has undertaken research stays at institutions like the University of Texas, University of Aveiro, and CentraleSupelec, with 35 short visits to several universities across Europe, Asia, and Africa. She has delivered 68 talks at international conferences, including 4 plenary sessions, and 11 invited presentations and seminars at scientific institutions. She serves on the editorial committees of several journals, including Applied Mathematics in Science and Engineering, Mathematics and Computers in Simulation, Fractal and Fractional, Computational and Applied Mathematics, and Communications in Nonlinear Science and Numerical Simulation. Google Scholar Profile | ResearchGate Profile

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Invariant graphs and dynamics in a family of piecewise linear maps

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Abstract

Invariant sets are very important objects in the study of dynamical systems. Its significance is clear in the case of systems possessing a first integral (an invariant), but this is also the case of highly dissipative systems as the one that we will consider in the talk: the family of piecewise linear maps of the form

$$F(x,y) = (|x| - y + a, x - |y| + b)\,,$$

where $(a,b) \in \mathbb{R}^2$.

Years ago, Grove and Ladas [5] introduced the family of maps $G(x,y) = (|x| + \alpha y + \beta, x + \gamma|y| + \delta)$ with $\alpha, \beta, \gamma \in \mathbb{R}$ and $\delta \in \{-1,0,1\}$, with the aim of generating a broader framework for studying generalized Lozi-type maps. These kind of maps were, in turn, introduced in the late 70ies by R. Lozi [6]. A particular case, known as the Gingerbread map, was studied in the 80ies by R. Devaney, [4].

The family of maps F intersects the general family of Grove and Ladas. In the last years, some works have appeared analyzing different particular cases of the Grove-Ladas family, see for example [1,7,8], just to cite some works that include subcases of the family F. Essentially, these works characterize cases in which all orbits converge to fixed points, periodic orbits, or are eventually-periodic. In fact, our motivation for the study of F was I. Bula's talk at the Sarajevo's 26th ICDEA, [2]. In that talk it was expressed the conjecture that for $a, b < 0$ all orbits are eventually periodic. As we will see, the global dynamics of the family F is substantially richer and the invariant sets of the map play an important role.

First, we prove that for $a \geq 0$ all the orbits are eventually-periodic and moreover the set of periodic orbits has finite cardinality:

Theorem A *If $a \geq 0$ then F has trivial dynamics. Moreover the set of periodic points of F has finite cardinality.*

The interesting situation occurs when $a < 0$. For this case, we prove that for all $b \in \mathbb{R}$ *there exists a compact invariant graph* Γ, such that the orbit of all $(x,y) \in \mathbb{R}^2$ (except perhaps a fixed points or some periodic orbits that are well characterized) converges to Γ in a finite number of iterates.

Theorem B *Set $a = -1$. For all $b \in \mathbb{R}$ there exists a compact graph Γ which is invariant under the map F such that for all $\mathbf{x} = (x,y) \in \mathbb{R}^2$ there exists $n_{\mathbf{x}} \in \mathbb{N}$ (that depends on $\mathbf{x}$) such that $F^n_{\mathbf{x}}(x,y) \in \Gamma$.*

From the dynamical viewpoint, and among other results, for each $b \in \mathbb{R}$ we characterize when $F|_\Gamma$ has positive or zero *topological entropy.*

Theorem C *Set $a = -1$. For each $b \in \mathbb{R}$ consider the map F restricted to the corresponding invariant graph Γ. Then*

(a) For $b \in (-\infty, -112/137] \cup [-1/36, 603/874] \cup \{1\} \cup [8, \infty)$ the map $F|_\Gamma$ has zero entropy.

(b) For $b \in [-13/16, -1/36) \cup [563/816, 1) \cup (1, 8)$ the map $F|_\Gamma$ has positive entropy.

(c) For $b \in [-112/137, -13/16] \cup [603/874, 563/816]$ *is non-decreasing in* b*, and there is a value of* b *in each of the above intervals, so that the entropy is zero in one of its subintervals and positive in the other.*

The results of this talk have been obtained in collaboration with Anna Cima, Armengol Gasull and Francesc Mañosas, and can be found in [3].

References:

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Biography

Víctor Mañosa is Associate Professor at the Department of Mathematics and the Institute of Mathematics (IMTech) of the Polytechnic University of Catalonia UPC-BarcelonaTech. He is member of the Dynamical Systems Group-UPC and long term collaborator of the Dynamical System Group at Universitat Autònoma de Barcelona (UAB).

His research is focused on the study of Dynamical Systems in a broad sense. Mainly, he has worked on the analysis of discrete systems and differential equations, including among other topics, the study of periodic orbits; stability and control problems; the modelling of systems with hysteresis; and the study of travelling wave solutions of PDE.

In the field of discrete dynamics, his interests have focused on questions such as the algebraic aspects of the integrability of discrete systems; the development of computational techniques for the analysis of periodic orbits; or the use of certain differential equations (Lie symmetries) for the study of nonlinear maps.

In 2015, along with Profs. A. Cima and A. Gasull, received the award for best JDEA article, from the International Society of Difference Equations, for the paper: *Basin of attraction of triangular maps with applications.*

Hybrid Discrete-Continuous Epidemic Models

Maia Martcheva

University of Florida

Abstract

Discrete models are used for species with non-overlapping generations while continuous ODE models are best suited for species with continuous births and deaths. When species with discrete and continuous generations interact the most adequate mathematical model is a hybrid discrete-continuous ODE model. In this talk I would introduce such a novel class of models and the analytical techniques for studying of such models. The techniques are used to compute the reproduction number of the hybrid system. It is shown that the disease-free equilibrium is locally asymptotically stable if the reproduction number is less than one and unstable, if the reproduction number is grater than one. Numerical methods for computation with such models are introduced and used to simulate the system.

Biography

Maia Martcheva is a professor of mathematics at University of Florida. She obtained her PhD at Purdue University in 1998. After that she was a postdoc at the Institute for Mathematics and its Applications, University of Minnesota, Arizona State University and an NSF Advance Fellow at Cornell University in 2002-2003. Since 2003 she has been an Assistant, Associate and Full Professor at the

Department of Mathematics, University of Florida. Maia Martcheva has published over 140 articles. Maia Martcheva has also published 3 books: Gender Structured Population Modeling (2005, SIAM), An Introduction to Mathematical Epidemiology (2015, Springer), and Age Structured Population Modeling (2020, Springer). Her research has been supported by the National Science Foundation. In 2016-2018 Maia Martcheva was a Managing Editor of Journal of Biological Systems. Currently, she serves on the editorial boards of Journal of Biological Systems, Journal of Biological Dynamics, Journal of Difference Equations with Applications, and Mathematics in Medical and Life Sciences

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Admissibility and Asymptotic Behaviors of Dynamical Systems: from Global Methods to Ergodic Theory Approaches

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Abstract

The admissibility methods represent some of the most important class of tools in exploring the asymptotic behaviors of dynamical systems. Their impact is due to both their effectiveness in describing various phenomena of uniform or nonuniform nature (such as stability, expansivity, dichotomy, trichotomy) through adequate solvability conditions imposed to certain input-output systems as well as due to their spectacular applications in control theory. In this lecture, we will present new admissibility methods for detecting the nonuniform/uniform asymptotic properties of discrete dynamical systems, that employ several representative classes of sequence spaces (invariant under translations or not), beginning with global conditions that take place on all the points of the parameter space and arriving to local-type conditions that are satisfied only on some subsets of positive measure. Starting from the techniques and results developed in [1-4], we present complete

characterizations for both stability and expansiveness of discrete variational systems from three perspectives, as follows: in each case we discuss a global method and two local approaches (that rely on ergodic theory arguments) and we also highlight several new applications. In the first part, we provide input-output conditions for nonuniform and uniform exponential stability by employing input and output spaces from three main classes of sequence spaces. In the second part, we introduce several notions of exact admissibility to explore the nonuniform and uniform exponential expansiveness and in this aim we use three specific classes of sequence spaces. For both stability and expansiveness, besides giving existence criteria, we discuss the hypotheses and the minimal requirements on the underlying input or output spaces via relevant examples. After that, we present applications to the study of the stability and expansiveness of continuous-time dynamics described by skew-product semiflows, by using nonuniform admissibility conditions with pairs of function spaces. Finally, inspired by some methods introduced in [5] we point out open problems, new directions and we discuss several future aims.

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Biography

Adina Luminița Sasu graduated BSc (1998) and MSc (1999) from the Faculty of Mathematics, West University of Timisoara, Romania, as a Valedictorian. She received the PhD degree in Mathematics with special distinction in 2002, with the PhD thesis entitled *The Asymptotic Behavior of Linear Skew-Product Flows*. In 2008 she was awarded the Prize "*In Hoc Signo Vinces*" for scientific activity, from the National Council of Scientific Research in Higher Education. In 2014 she obtained the Habilitation in Mathematics, with the habilitation thesis defended at University Babes-Bolyai Cluj-Napoca, thus becoming a PhD supervisor in Mathematics. Since 2016, she has been Full Professor at Faculty of Mathematics and Computer Science, West University of Timisoara, Romania. Since 2012 she has been a member of the Senate of West University of Timisoara and since 2016, she serves as President of the Research Commission of the University Senate. From 2016 to 2024 she was the director of the Doctoral School of Mathematics from West University of Timisoara. Since 2019 she has been a member of the Academy of Romanian Scientists. Her scientific interests cover topics concerning the asymptotic behavior of dynamical systems, qualitative properties of evolution equations in discrete and continuous time, and respectively control theory. She published over 85 articles in the areas of Mathematics and Applied Mathematics, some of the main subjects of her works being devoted to stability, dichotomy, trichotomy, admissibility, robustness, and various control techniques. She also published two mathematical monographs and three lecture notes for undergraduate, master and PhD students. Since 2005 she has been a member of *the International Society of Difference Equations.* She organized the *23rd International Conference on Difference Equations and Applications - ICDEA 2017* as Chair of the Organizing Committee (https://icdea2017.uvt.ro/).

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Discontinuous maps and their applications: Exploring the bifurcation structures

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Abstract

Nonsmooth dynamical systems, in particular, piecewise smooth maps, are currently actively studied by many researchers from various theoretical and applied fields. Quite often, the main focus is on the study of possible attractors, their basins, as well as the bifurcations that these attractors may undergo. In the parameter space, the existence regions of various attractors can form bifurcation structures that significantly differ from those observed in the smooth case, mainly due to border collision bifurcations occurring when an attractor collides with a border separating different definition domains of the system. The most studied are bifurcation structures associated with 1D piecewise smooth maps, continuous and discontinuous, defined

on two partitions (see, for example, Avrutin et al (2019) where bifurcation structures related to attracting cycles and chaotic attractors of 1D piecewise monotone maps are described in detail). Among 2D nonsmooth maps, it is worth mentioning the continuous piecewise linear map known as 2D border collision normal form, which for several decades continues to attract the attention of researchers. Many important results have been obtained, but a complete description of the bifurcation structure of the parameter space of this map is still missing. This talk aims to recall some known results on various bifurcation structures in discontinuous maps and to present new results related to 2D discontinuous piecewise linear maps defined on three partitions. The study of such maps is interesting from a theoretical point of view, and is additionally motivated by their application, for example, in modeling the dynamics of financial markets with heterogeneous agents (see, e.g. Sushko and Tramontana (2024)).

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[2] I. Sushko, F. Tramontana. Regular and Chaotic dynamics in a 2D discontinuous financial market model with heterogeneous traders. Mathematics and Computers in Simulation, 2024.

Biography

Iryna Sushko, applied mathematician, live in Kyiv (Ukraine). She received the MS degree in applied mathematics from the Kyiv State University, Faculty of Cybernetics, in 1989, and just after joined a post-graduate course at the Institute of Mathematics, National Academy of Sciences of Ukraine. Scientific supervisors Prof. A. Sharkovsky and Prof. Yu. Maistrenko introduced Iryna to the nonlinear dynamical systems theory, which then became her main research field. Iryna received the PhD in 1993, and since this year she is appointed to a research position in the Institute of Mathematics, Department of Differential Equations and Oscillation Theory. In 2005-2006 Iryna was affiliated to the University of Urbino, Italy, due to Marie Curie Fellowship. In fact, this university has become her second academic home thanks to frequent research stays there and active collaboration with Laura Gardini, Fabio Tramontana, Gian Italo Bischi, Davide Radi and other colleagues from this university. Since 2010, Iryna has also been affiliated with the Kyiv School of Economics as a visiting professor and researcher. Current research interest of Iryna is related to piecewise smooth discrete-time dynamical systems, continuous and discontinuous, bifurcation structures of their parameter space, applications in economics, finance, etc., computer simulation and numerical experiments. Iryna is the co-author of more than 120 scientific papers, Associate Editor of two Elsevier Journals (MATCOM and CNSNS), member of scientific and organizing committees of many conferences.

The National Science Foundation

Amina Eladdadi
Department of Mathematics The College of Saint Rose

A brief presentation of funding opportunities related to the NSF Division of Mathematical Sciences (DMS) Programs and international collaborations, followed by a Q&A with NSF Program Director.

Biography

Dr. Amina Eladdadi
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Currently a Visiting Scientist and Program Director at the Mathematical Biology Program in the Division of Mathematical Sciences at the National Science Foundation.

Speakers

The Influence of Periodic Reproduction and Predator Evolution on the Dynamics of a Predator-Prey Model

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Abstract

We extend the predator prey model developed in Ackleh et. al (2019) to account for periodic reproduction in the prey due to seasonality as well as evolution in the predator to resist toxicant. We first model the reproduction as a periodic function of period-two and show the predator-prey model attains an interior 2-cycle. We study the local asymptotic stability of the periodic solution and establish persistence of the system and compare our results with those in Ackleh et al. (2019), which assumed that reproduction is continuous. We find that, while periodic reproduction may be detrimental to the prey species in isolation, it becomes advantageous for large inherent reproduction values of the prey in the presence of the predator. Then we model the predator evolution to resist the effect of toxicants, assuming a trade-off between toxicant resistance and the ability of the predator to capture prey and survive. We study the dynamics of the resulting predator-prey model and compare the results with those in Ackleh et al. (2019), which assumed instead that the prey evolves to resist toxicants.

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How Do Prey Dynamics Impact Predator-Prey Interactions?

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Abstract

We develop variations of a discrete-time predator-prey model to examine how intrinsic properties of the prey population may impact overall system dynamics. We focus on two properties of the prey species, namely developmental stage structure and undercompensatory (contest competition) versus overcompensatory (scramble competition) density dependence. Through analysis of these models, we examine how these different prey features affect system stability. Our results show that when prey growth is overcompensatory, the predator may have a stabilizing effect on the system dynamics with increasing predator density reversing the period doubling route to chaos observed with Ricker-type nonlinearities. Moreover, we find that stage structure in the prey does not have a destabilizing effect on the system dynamics unless the prey projection matrix is imprimitive or close to imprimitive, as may occur for semelparous species. In this case, a sufficiently large predator density may stabilize cycles that are otherwise unstable.

Shortest Closed Billiard Orbits On Convex Tables

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Abstract

Given a planar compact convex billiard table T , we give an algorithm to find the shortest generalised closed billiard orbits on T . (Generalised billiard orbits are usual billiard orbits if T has smooth boundary.) This algorithm is finite if T is a polygon and provides an approximation scheme in general. As an illustration, we show that the shortest generalised closed billiard orbit in a regular n-gon Rn is 2-bounce for $n \geq 4$, with length twice the width of Rn. As an application we obtain an algorithm computing the Ekeland–Hofer–Zehnder capacity of the four-dimensional domain $T \times B2$ in the standard symplectic vector space $R4$. Our method is based on the work of Bezdek–Bezdek in (6) and on the uniqueness of the Fagnano triangle in acute triangles. It works, more generally, for planar Minkowski billiards.

On the generating function space of a Volterra difference equation with infinite delay

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Abstract

The talk is devoted to the study of the generating function space, G, of a Volterra difference equation with infinite delay.

The first goal is to characterize the Volterra equations for which the space G is finite-dimensional, as well as to present, in terms off Hankel matrices, explicit formulas to compute dimension of G.

The second goal concerns the characterization of the space G, when G is finite-dimensional. It is proved that, in this case, the space G is completely characterized by its dimension and by the generating function space of a linear equation of finite order whose coefficients can be obtained through the analysis of another Hankel matrix.

These results play an interesting role in the characterization of the solutions of a Volterra difference equation and show that the dimension of G can be seen as a measure of how much the solutions of the Volterra equation differ from the solutions of a linear difference equation with finite order.

Advances in Stochastic Modeling of MERS Corona: Mathematical and Computational Innovations

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Abstract

The biological modelling of disease transmissions among humans, plants, and animals is playing a vital role in their optimal solutions and future predictions with necessary assumptions for the eradication of the problems. In this article, a novel stochastic model is constructed with the use of brownian and levy noises. Besides the applications of the additions, we have considered treatment and media roles in the awareness of public and adaptation of necessary measurements. The model is basically representing MERSE Corona transformation from human to camels and from the camels to humans. For this, we have considered the existing population of a region in the susceptible, infected, and recovered classes. For both populations, we have developed R0 and have analysed that until the R0 is greater for either class, the disease is spreading. For the complete eradication, we need to produce a suitable environment that gives us the values of R0 less than 1 for both populations. Computational scheme and illustrative examples have further verified that the disease can be eradicated subjected to the values of R0.

Advances in Stochastic Modeling of MERS Corona: Mathematical and Computational Innovations

Amira Khelifa

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Abstract

In this work we solve the following system of difference equation

$$x_{n+1}^{(j)} = \frac{F_{m+2} + F_{m+1} x_{n-k}^{((j+1) \mod (p))}}{F_{m+3} + F_{m+2} x_{n-k}^{((j+1) \mod (p))}}, \quad n, m, p, k \in \mathbb{N}_0, \quad j = \overline{1, p},$$

where $(F_n)_{n \geq 0}$ is the Fibonacci sequence. We give a representation of its general solution in terms of Fibonacci numbers and the initial values. Some theoretical justifications related to the representation for the general solution are also given.

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Ulam stability and instability for n-periodic dynamic equations on isolated time scales

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Abstract

We apply a new definition of periodicity on isolated time scales introduced by Bohner, Mesquita, and Streipert to the study of Ulam stability. If the graininess (step size) of an isolated time scale is bounded by a finite constant, then the linear $\omega-$periodic dynamic equations are Ulam stable if and only if the exponential function has modulus different from unity. If the graininess increases at least linearly to infinity, the $\omega-$periodic dynamic equations are not Ulam stable. Applying these results, we give several interesting examples of first-order linear $1-$periodic or $2-$periodic dynamic equations on specific isolated time scales such as h-difference equations, q-difference equations, triangular equations, Fibonacci equations, and harmonic equations; in some cases the minimum Ulam stability constant is found.

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Study and Chaos Control on Discrete Dynamics of Cournot Triopoly Team-Game

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Abstract

In this study, we explore the dynamics of a nonlinear discrete-time triopoly game involving firms with bounded rationality. We use a system of three difference equations to model the game. Our analysis examines the existence and stability of equilibrium points, with a specific focus on the Nash equilibrium point. We find that the two routes to chaos are flip and Neimark-Sacker bifurcations. Moreover, an increase in adjustment speed destabilizes the system, leading to more complex dynamic behavior. This behavior is examined through numerical simulations, where various parameters are adjusted to observe their effects. State feedback control effectively stabilizes the system at the Nash equilibrium point. This control approach delineates three stability boundaries, defining a triangular region within the parameter space. Each boundary line corresponds to specific scenarios influencing overall stability, with intersections marking the stability region.

Discrete-time Replicator Equations on Wardrop Optimal Networks

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Abstract

In 1952, J. Wardrop [1] formulated two equilibrium principles of optimal flow distributions in networks that describe the *user (Wardrop-Nash) equilibrium* and *system optimum*. Later, Dafermos and Sparrow [2] coined the terms “user-optimized” and “system-optimized” to distinguish between the Nash equilibrium where users act unilaterally in their own self-interest versus when users are forced to select the strategies that optimize the total network efficiency. Recently, there has been an increasing interest in the study of inefficiency of Nash equilibrium problems in noncooperative games [3]. The degradation in a system’s efficiency due to selfish non-cooperative behavior of agents in the system is called the *price of anarchy*.

In this talk, the concept of a *Wardrop optimal network* in which the selfish behavior of noncooperative network users has a negligible impact on network performance degradation is introduced. These networks admit Wardrop optimal flows that are both user equilibrium and system optimum, and are the only networks for which the price of anarchy is exactly equal to its least value 1. We investigate dynamic properties of Wardrop optimal networks and examine the Wardrop optimal flows on dynamic networks in which cost functions that generate the network change over time, i.e., at each next time instant (iteration, observation) the functions may differ from those at the previous step. We present a novel dynamical model for optimal flow distribution on Wardrop optimal networks, using the ideas of evolutionary game theory, which unlike the classical game theory, focuses on the dynamics of strategy change. The primary way to study the evolutionary dynamics [4] is through replicator equations that show the growth rate of the proportion of agents using a certain strategy and that rate is equal to the difference between the average payoff of that strategy and the average payoff of the agents population as a whole. The key idea is that replicators whose fitness is larger (smaller) than the average fitness of population will increase (decrease) in numbers. The dynamic stability analysis of stationary solutions of replicator equations complement the static approach to evolutionary games.

Our dynamical model is based on discrete-time replicator equations generated by nonlinear similar-order preserving mappings [5, 6]. In particular, we consider the discrete-time replicator dynamical systems generated by convex differentiable functions [7], and by Schur potential functions, namely, by gradient vector fields of Schur-convex functions, which wll also be exemplified by employing complete symmetric functions and gamma functions as those that generate the replicator dynamics. The equilibrium, convergence to fixed points, and asymptotic stability conditions of the replicator equation dynamics are studied. For the proposed discrete dynamical system, the Nash equilibrium, the Wardrop equilibrium, and the system optimum represent the same point in the state space, i.e., flow in the network. We

also discuss the results of computational experiments, where, *inter alia*, we analyze the convergence rate of orbits of replicator dynamical system to fixed points for different types of functions that generate the replicator equation.

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A class of planar population maps

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Abstract

As early as 1976 Hassell and Comins studied the dynamics, and in particular competitive exclusion, of a class of discrete time population models where the per-capita growth rate was a function of a linear sum of 2 populations:

$$(x, y) \to \phi(xf(x + \alpha y) - b, yg(y + \beta x) - b) \tag{1}$$

where $\alpha, \beta, b > 0$ and $f, g : R^+ \to R^+$ are continuously differentiable functions. They referred to these models ϕ as competitive, but the not necessarily competitive in the language of monotone systems theory (i.e. they are not necessarily retrotone). Niu and Ruiz-Herrera have also considered models of the form (1) (with b = b = -1), taking a new approach that uses translation arcs. Their models need not be retrotone, although their map ϕ does need to be an orientation-preserving homeomorphism. Here we extend our Lyapunov function approach of the Ricker map (where $f(z) = er - z, g(z) = es - z$) to maps (1) with more general f, g and which are not necessarily retrotone, nor homeomorphisms.

Population models for the protection of ecosystem services

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Abstract

Understanding how and why populations are regulated is a central concern in applied ecology. Mathematical models, especially those parameterized with field- and laboratory derived data, are a powerful tool for understanding population dynamics and the forces governing population regulation. I describe here an application of simple population models applied to the concept of surrogate species in the environmental sciences. The surrogate species concept is often used in conservation science in order to understand risks faced by endangered species or economically important species, especially those that are important providers of ecosystem services. Surrogates are often chosen on the basis of convenience or similarities in physiology or life history to species of concern, but few formal protocols exist for evaluating the choice of surrogates. Furthermore, our ability to predict how species of concern will fare when subjected to anthropogenic disturbances such as environmental contaminants or toxicants is often based on potentially misleading comparisons of static toxicity tests (e.g., the LC50). Here I present an alternative approach that features matrix models parameterized with life history data, applied to different assemblages of species. I describe several case studies using data from diverse taxa including endangered salmonids and a suite of parasitoid wasps important for biological control in agroecosystems to illustrate how we can incorporate life history traits into models in order to better understand and predict population outcomes. The results demonstrate that we cannot always reliably use the response of designated surrogate species to predict the fate of similar – even closely related – species exposed to the same disturbances. This modelling approach reveals implications for how we assess risk and set conservation policy in both managed and natural/semi-natural ecosystems.

Perturbation theory of angular spectra

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Abstract

We consider a nonautonomous linear dynamical system in discrete time

$$u_{n+1} = A_n u_n, \quad n \in \mathbb{N}_0, u_n \in \mathbb{R}^d, A_n \in \mathbb{R}^{d,d} \tag{2}$$

and its angular spectrum Σ_s of dimension $s \in \{1, \dots, d\}$ as defined in [2]. The set Σ_s comprises all accumulation points of longtime averages

$$\alpha_n(V_0) = \frac{1}{n} \sum_{j=1}^{n} \measuredangle(V_{j-1}, V_j), \quad n \in \mathbb{N},$$

where $V_j = A_{j-1}V_{j-1}$ for $j \geq 1$ and V_0 ranges over all s-dimensional subspaces of $\mathbb{R}^d$. The symbol $\measuredangle(\cdot,\cdot)$ denotes the maximal principal angle between two subspaces. Our main result states that the angular spectrum Σ_s stays invariant if the perturbations E_n of the perturbed matrices $A_n + E_n$ are absolutely summable, and if the system (2) has a so-called complete exponential dichotomy (CED). The CED notion generalizes the requirement of all eigenvalues being semisimple in the autonomous case, to the nonautonomous case. We present an application of this result to the linearization of a 3-dimensional system of Hénon type with a Shilnikov homoclinic orbit.

Then we consider perturbations of (2) which are not summable, in particular parametric perturbations. For some model examples we show that upper semicontinuity w.r.t. the Hausdorff metric holds but lower semincontinuity generally fails. If time permits we will discuss some corresponding results for continuous time systems for which the appropriate notion of angular spectrum is based on the theory in [1].

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Discrete Dynamical Systems with Memory in Social Sciences

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Abstract

The time evolution of a social system is often represented by a discrete dynamical system in the form

$$x(t+1) = f(x(t), g(x(t)) \tag{3}$$

where $x \in \mathbb{R}$ is the dynamic variable that depends on agents' decisions, $g : \mathbb{R} \to \mathbb{R}$ is a function representing an index of performance (i.e. gain, fitness, profit or something similar). This model assumes that the agents' decisions about the next period strategy are based on the knowledge of current performance only, whereas in many social and economic systems past performances are considered as well, a sort of memory or accumulated gains. So a weighted average of the gains observed in the current and past time periods is considered, i.e.

$$x(t+1) = f(x(t), U(t)) \tag{4}$$

where

$$U(t) = \sum_{k=0}^{M} \omega_k g(x(t-k)) \tag{5}$$

that is, the agents decide their next strategy according to a moving average of the performances observed during the more recent M time periods, where M is the length of memory and ω_k are normalized weights, $\sum_{k=0}^{M} \omega_k = 1$. For $M = 0$ the case with no memory is obtained, and for $M > 0$ the distribution of weights can be used to modulate the "shape" of past memory. The model becomes a difference equation of order $M+1$, equivalent to a $M+1$ dimensional discrete dynamical system, hence the introduction of a finite memory gives rise to a higher dimensional dynamical system. An alternative consists in considering a discounted sum of all the performances along the whole story of the dynamical system, obtained by taking, at each time step, a convex combination of the current performance and the accumulated average computed in the previous time period:

$$U(t) \quad = (1-\omega)\, g(x(t) + \omega U(t-1) \tag{6}$$

with $\omega \in [0, 1]$, $U(0) = 0$. By backward induction reasoning it is easy to get

$$U(t) = (1-\omega) \sum_{k=0}^{t-1} \omega^k g(x(t-k)) + \omega^t U(0)$$

a discounted measure along the previous performance history expressed by a weighted sum with exponentially fading weights. Again, the parameter $\omega \in [0, 1]$ gives a measure of the memory, as $U(t) = g(x(t))$ for $\omega = 0$, whereas the uniform arithmetic mean of all the performances observed in the past is obtained in the other limiting case $\omega = 1$. If the recursive scheme (6) is plugged into the dynamic model (4) then we get the two-dimensional dynamical system

$$\begin{aligned} x(t+1) &= f(x(t), U(t)) \\ U(t+1) &= (1-\omega)\, g(f(x(t), U(t))) + \omega U(t) \end{aligned} \tag{7}$$

The fixed points of the map (3) are also fixed points of the maps with memory (5) and (5), so it is interesting to study how their stability properties are modified by the presence of memory.

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Discrete algorithms for approximating spectral optimal partitions

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Abstract

Partitions minimizing functionals depending on the eigenvalues of the Dirichlet-Laplace eigenvalues arise in different applications in population dynamics, modelization of chemical reactions and community detection in graphs. Theoretical tools do not allow the complete characterization of such optimal partitions, motivating the need for efficient numerical tools allowing their approximation. More details and references can be found in [1].

The problem of interest is finding partitions $(\omega_i)_{i=1..n}$ of a domain D, a subset an euclidean space or of a manifold, which minimize spectral functionals depending on the spectrum of the Dirichlet Laplace operator. The main examples are

$$\mathcal{F}(\omega_i) = \lambda_1(\omega_1) + ... + \lambda_1(\omega_n) \quad \text{and} \quad \mathcal{F}(\omega_i) = \max_{i=1..n} \lambda_1(\omega_i).$$

Shapes are relaxed to density functions taking values in $[0, 1]$ and the Laplacian is discretized using finite differences, reformulating the above problems in a finite dimensional context, which allows efficient implementations.

The algorithms obtained allow to find numerical approximations of optimal partitions for a large class of domains D in $\mathbb{R}^2, \mathbb{R}^3$ and on surfaces in $\mathbb{R}^3$. The numerical algorithm is inspired by [1] and more details about the contents of this talk can be found in [2-3]. The algorithm also admits straightforward generalizations to graphs, leading to efficient graph partitioning algorithms.

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Continuous-time difference equation with two interfering delays: Prescribed stabilization of some hyperbolic PDE's

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Abstract

This talk addresses the stability of continuous-time delay-difference equations with two interfering delays. Thanks to some standard spectral methods, we emphasize the link of the characteristics of such a class of dynamical systems with the ones resulting from the control of some hyperbolic PDEs. A control method is proposed, which is merely a delayed output feedback relying on a partial pole placement idea, consisting in the prescription of an appropriate exponential decay rate to the closed-loop system solution. A lower-bound of the spectral abscissa of the resulting characteristic equation is obtained. The boundary control as well as the pointwise control of the wave equation illustrate the interest of the obtained results.

Stability and instability results for fractional-order differential equations with Prabhakar derivatives

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Abstract

Necessary and sufficient conditions are explored for the asymptotic stability and instability of linear two-term fractional-order differential equations with Prabhakar derivatives of the following type:

$$D^{\gamma_1}_{\alpha,\beta_1,\omega}y(t) + aD^{\gamma_2}_{\alpha,\beta_2,\omega}y(t) + by(t) = 0,$$

where $\omega < 0$, $\gamma_i > 0$, $\alpha, \beta_i \in (0, 1]$ with $\beta_1 - \alpha\gamma_1 \geq \beta_2 - \alpha\gamma_2 \geq 0$, for $i \in \{1, 2\}$.

By means of the Laplace transform method, we obtain the following characteristic equation:

$$s^{\beta_1-\alpha\gamma_1}(s^\alpha - \omega)^{\gamma_1} + as^{\beta_2-\alpha\gamma_2}(s^\alpha - \omega)^{\gamma_2} + b = 0.$$

Firstly, we characterize the fractional-order dependent stability and instability properties of the considered equations. Then, we obtain necessary and sufficient

conditions for the stability and instability of the equations, for any fractional orders, in terms of the characteristic parameters a and b. Numerical simulations are also provided in order to illustrate our theoretical results.

Future research directions will focus on the discrete counterparts of the fractional differential equations with Prahakar derivatives and applications. Due to the versatility, memory retention capabilities and improved accuracy of fractional-order derivatives, we emphasize the advantages of considering fractional operators when modeling real world phenomena.

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Behavior of Systems of Piecewise Linear Difference Equations with Many Periodic Solutions

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Abstract

Ladas posted an open problem about the generalized Lozi map and Gingerbreadman maps to a system that were mentioned in [2]:

$$\begin{cases} x_{n+1} = |x_n| + ay_n + b, \\ y_{n+1} = x_n + c|y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2,$$

where parameters a, b, c and d are in $\{-1, 0, 1\}$.

In our presentation we analyse behavior of system

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2, \quad b > 0,\ d > 0. \tag{2}$$

If $2d - b \leq 0$, then there exist an unstable equilibrium $(\frac{-2b-d}{5}, \frac{2d-b}{5})$. It is proved that there are no solutions with period 2, but depending on the values of parameters

b and d there are solutions with periods 3, 4, 7, 10, 11, 14. It is possible that other periodic solutions can be found. A similar behavior exists for the system with $-d$ ([1]). In some cases, several cycles exist simultaneously. Numerical experiments show that all solutions converge to some cycle.

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Ulam Stability of a Second Linear Differential Operator with Nonconstant Coefficients

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Abstract

In this lecture we will present a result on Ulam stability for a second order differential operator acting on a Banach space. This outcome is connected to the existence of a global solution for a Riccati differential equation as well as to some appropriate conditions on the coefficients of the operator.

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The dynamics of the γ-Ricker model of order two

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Abstract

We consider the population model given by the order two difference equation

$$x_{n+1} = a x_n^{\gamma} e^{-x_n - \rho x_{n-1}}, \tag{8}$$

where $x_n \geq 0$, $a > 0$, $\rho \geq 0$ and $\gamma \geq 1$. When $\gamma = 1$ and $\rho = 0$, we obtain the classical Ricker model [5]. The parameter γ was introduced in [1] as cooperation in the population and produces the so-called Allee effect, giving the extinction of the species when the population is small enough [3]. An extension of the Ricker model to order two was proposed, for instance, in [2,4] in which the parameter $\gamma = 1$. Here, we introduce the cooperation parameter γ to be greater than one and study the existence of the Allee effect for the model. We explore the local and global dynamics of the model.

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Postcards from the edge: Technological competition and market dynamics

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Abstract

This research provides a fresh insight into the interplay among technology, innovation, firm goals, and market competition. Utilizing the theory of incomplete contracts, it explores how technological advancements can fundamentally transform the essence and objectives of a firm. A discrete-time duopoly model is introduced to examine how firms with bounded rationality compete technologically, underscoring the implications of excessively prioritizing a so-called *critical resource*. Specifically, the emergence of complex dynamics is addressed within various market conditions: (i) when firms employ symmetric decision-making processes; (ii) when firms utilize diverse decision-making strategies; (iii) when firms alter their decision-making approaches based on their market share relative to competitors. Economically, the study concludes that while technology serves as a significant tool for gaining a competitive edge, it does not automatically guarantee market dominance. This finding is vital in an environment where technological superiority is often equated with market success.

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Parametrized Jacobian Conjeture

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Abstract

The Jacobian Conjecture states that if a polynomial function $P\colon \mathbb{K}^n \to \mathbb{K}^n$; where $\mathbb{K}$ is a field with characteristic zero; has a constant and non null Jacobian determinant, then P has a polynomial inverse. This conjecture was introduced by O.H. Keller in 1939 in [2] and it is still open even in dimension two.

The Bass-Connell-Wright/Yagzhev Reduction Theorem (see [1, 3] respectively) states that studying the Jacobian conjecture for maps of the I+H form with nilpotent JH is enough. In this talk, we will present a parameterized version of the conjecture and provide a sufficient result to satisfy this version. For this purpose, a useful tool is a problem on global stability, which assumes that the spectrum of exponential dichotomy belongs to $(-\infty, 0)$.

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Dynamical analysis of an OLG model with interacting epidemiological and environmental domains

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Abstract

We propose and study an economic-epidemiological-environmental model based on a three-dimensional nonlinear dynamical system. The dynamics of epidemics are driven by an SIS model, in which the transmission rate of epidemics negatively depends on the share of public expenditures devoted to healthcare and positively on the pollution level in the environment. The economic domain is described by an OLG model with capital accumulation, in which in household preferences the psychological discount factor is a function that positively depends on the number of susceptible agents, which supply labor with a total factor productivity that decreases as the number of infected people increases. The environmental situation is described by the pollution level, which increases due to the emissions during the production process, naturally decreases and can be abated by adopting a suitable technology that is supported by a share of resources collected from the taxation of the produced output. Our initial focus is on the steady states that are characterized by the presence (endemic steady states) or absence (disease-free) of infected agents. Analyzing the nonlinear functions characterizing the interactions between the economic, the epidemiological and the environmental spheres, we provide comparative statics for the capital level, the share of susceptible agents and the pollution stock. Furthermore, we study steady-state stability, focusing on the potential dynamics that may arise when stability is lost.

The 3x+1 Problem; a discrete dynamics approach

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Abstract

The $3x + 1$ Problem is a well-known open problem. Consider the map T defined as $T(n) = n/2$ if n is an even integer and $T(n) = 3n + 1$ if n is an odd integer. The conjecture states that any positive integer n will eventually iterate under the map T to the number one. Hundreds of papers and books have been written about this problem since the 1970s, but it seems impenetrable. In this talk, we explore the insight gained by extending the map to the real axis and using the tools of discrete dynamics.

Riordan Arrays and Generating Functions for Generating Trees

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Abstract

In this paper, we use an algebraic approach to study the relation between the structures of generating trees and Riordan arrays and we show how some well-known generating trees can be studied from the Riordan arrays point of view and we will present the methods for counting trees based on generating functions concerning tree enumeration, by which the counting results are obtained. The concept of generating trees has been successfully applied to other combinatorial classes such as lattice paths. Finally, we prove that the generating trees can be defined by taking into consideration of restricted lattice path problems.

Also, we study some lattice paths associated with lower triangular arrays. An infinite lower triangular matrix $F = (f_{n,k})_{n,k \in \mathbb{N}}$ is said to be associated to a generating tree with root (c) if $f_{n,k}$ is the number of nodes at level n with label $k + c$. Generating trees are fascinating structures in combinatorics that provide a way to encode the combinatorial objects through recursive construction which can be enumerated at the various levels of trees by counting the different labels. We illustrate our main results by several examples concerning classical combinatorial structures with respect to certain statistics.

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Existence of homoclinic solutions and ground state solutions for discrete nonlinear Schrödinger equations involving strongly indefinite potential

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Abstract

This article is a step towards the study of discrete nonlinear Schrödinger equation (DNLS in short) involving strongly indefinite potential functions. That is

$$-\Delta u(n) + V(n)u(n) - \omega u(n) = f(n, u(n)), \quad n \in \mathbb{Z}$$

with $\lim_{|n|\to\infty} |V(n)| = \infty$. The existence of so-called ground state solutions in discrete nonlinear Schrödinger equations with superlinear nonlinearity is obtained. In previous work, coercive potential functions, frequently assumed to be present, may not always be strongly indefinite. We work along this direction to address this gap and improve upon previous work. The arising difficulty is that the energy function of the DNLS equation induced by the strongly indefinite potential function V is unbounded from above and below both. To address this challenge, a strongly indefinite version of the compact embedding theorem is developed in this work. It is shown that the boundedness and linking structure of Cerami sequences remain even in this case. To the best of our knowledge, this is the first attempt to investigate a discrete nonlinear Schrödinger equation with a strongly indefinite potential function. And our results also improve some existing ones in the literature.

The dynamics of industry location and residence choice in the presence of pollution and commuting costs

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Abstract

In this paper we introduce commuting costs in a New Economic Geography (NEG) model where individuals make two different types of choices regarding their place of residence and their place of work. In a two-region setting, the residence choices are determined by the utility difference between the two regions. Households have an incentive to migrate to the region where they enjoy a higher utility relative

to the average utility of the economy, where this utility depends (positively) on the availability of consumption goods and (negatively) on local pollution; while commuting decisions (concerning individual family members) are determined by the difference between the remuneration in the region to which individuals commute and the average remuneration of the economy. We introduce a policy intervention aiming to reduce commuting flows motivated by environmental concerns. This policy is equivalent to a reduction in the speed of adjustment of commuting. However, individuals can switch to a costly, but non-restricted commuting alternative. They will choose this option, if the wage differential is high enough - so the cost of the non-restricted alternative represents a threshold for the dynamics. We compare our analysis with the case of no commuting costs as explored in Commendatore et al. (2023). Preliminary results on the dynamics show that: i) when commuting costs are sufficiently high they do not impact on the dynamics and only the regulatory policy plays a role. The slow-down of the speed of adjustment of commuting affects the stability of the interior symmetric fixed point by increasing the Neimark Saker bifurcation value; it also affects the stability of the boundary Core-Periphery fixed points by increasing the bifurcation value for the border collision bifurcation; ii) when commuting costs are sufficiently small the dynamics hit the threshold passing through alternative regimes. Finally, we will try to identify stylized empirical facts regarding the relationship between regulatory policies such as traffic restriction, pollution and migration.

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Matrix-valued periodic functions on isolated time scales

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Abstract

A function f is called ω-periodic when for all $t \in \text{dom}(f)$, $f(t+\omega) = f(t)$, but this definition is not appropriate for time scales, which are in general not closed under addition. A definition of scalar periodic functions in quantum calculus was provided in [3] and later expanded to isolated time scales in [1-2]. In this work, we further generalize the definition for matrix-valued functions, investigate periodic solutions to systems of dynamic equations, and show an application of the extended definition to controllability of linear systems with periodic coefficients.

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An oligopoly model with three time delays

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Abstract

Necessary and sufficient conditions are explored for the asymptotic stability and instability of linear two-term fractional-order differential equations with Prabhakar derivatives of the following type:

$$D^{\gamma_1}_{\alpha,\beta_1,\omega}y(t) + aD^{\gamma_2}_{\alpha,\beta_2,\omega}y(t) + by(t) = 0,$$

where $\omega < 0$, $\gamma_i > 0$, $\alpha, \beta_i \in (0, 1]$ with $\beta_1 - \alpha\gamma_1 \geq \beta_2 - \alpha\gamma_2 \geq 0$, for $i \in \{1, 2\}$.

By means of the Laplace transform method, we obtain the following characteristic equation:

$$s^{\beta_1-\alpha\gamma_1}(s^\alpha - \omega)^{\gamma_1} + as^{\beta_2-\alpha\gamma_2}(s^\alpha - \omega)^{\gamma_2} + b = 0.$$

Firstly, we characterize the fractional-order dependent stability and instability properties of the considered equations. Then, we obtain necessary and sufficient conditions for the stability and instability of the equations, for any fractional orders, in terms of the characteristic parameters a and b. Numerical simulations are also provided in order to illustrate our theoretical results.

Future research directions will focus on the discrete counterparts of the fractional differential equations with Prahakar derivatives and applications. Due to the versatility, memory retention capabilities and improved accuracy of fractional-order derivatives, we emphasize the advantages of considering fractional operators when modeling real world phenomena.

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Discrete-time Darwinian Dynamics

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Abstract

In his seminal 1948 paper on matrix models, P. H. Leslie introduced a logistic version of his famous discrete time matrix model for the dynamics of an age-structured population [1]. After summarizing the global asymptotic dynamics of this model, I will derive a Darwinian (evolutionary game theoretic) version that accounts for evolutionary adaptation of an individual's fertility and survival rates. I will describe a theorem that accounts for the global dynamics of the resulting model and then discuss the notion of an evolutionary stable strategy (ESS). The modeling methodology and analysis will be illustrated by an example based on an adaptive trade-off between adult fertility and post-reproductive survival. This example also provides some biological punch lines concerning the life history traits of semelparity and iteroparity and their evolutionary stability or instability

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A Discrete Smoking Disease Model with Impact of Media and Awareness

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Abstract

Smoking is a leading cause of preventable death in many nations due to its adverse effects on many body organs, resulting in strokes, heart disease, and other respiratory disorders. Smoking can raise the risk of lung cancer in both men and women. Smoking increases the risk of getting cardiovascular diseases. Media campaigns, commercials, and quitting programs can significantly reduce smoking rates. This article develops a non-linear compartmental model to examine how media knowledge affects the spread of smoking among smokers and nonsmokers. With this motivation, a discrete mathematical model is framed and developed in this research work. In addition to this, the relapse population who temporary quit smoking back to becoming smokers and also the from smoker and temporary smokers to permanently quit smokers are analysed. The necessary conditions for the stability of equilibria in the discrete model are defined. Numerical simulations are performed to highlight the model's complex dynamics.

Hadamard Fractional Calculus on Nabla Time Scales

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Abstract

This article proposes a Hadamard-type nabla fractional sum using the theory of time scales. Then, we define a Hadamard-type nabla fractional difference and derive some of its fundamental properties, such as the power and composition rules. Finally, we consider a particular class of initial value problems for Hadamard-type fractional difference equations and obtain their equivalent fractional sum equations.

Dichotomies for Triangular Systems via Admissibility

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Abstract

We study the relationship between the exponential dichotomy properties of a triangular system of linear difference equations and its associated diagonal system. We use admissibility to give new shorter proofs of results obtained in Battelli et al. [1] and we also establish new necessary and sufficient conditions that the diagonal system have a dichotomy when the triangular system has a dichotomy. The talk is based on the results in [2].

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Global Dynamics of Evolutionary autnomous and Periodic Models

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Abstract

We investigate the global dynamics of discrete-time phenotypic evolutionary models, both autonomous and periodic. We developed the theory of mixed monotone maps. We applied it to show that the positive equilibrium of the autonomous evolutionary model of single and multi-species is globally asymptotically stable. Then we extend this result to the corresponding evolutionary model with periodic parameters.

Discrete Mathematical Models of Tuberculosis

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Abstract

Tuberculosis (TB) continues to pose a significant global health challenge, affecting millions of individuals worldwide. Its complex transmission dynamics and resilience across diverse populations highlight the critical need for an in-depth understanding of the disease and the development of effective strategies for its control. The application of mathematical modeling has become instrumental in exploring these dynamics and in formulating potential interventions. The conceptual underpinnings of these models trace back to the foundational work of Sir Ronald Ross, who, in 1911, developed the first mathematical model for malaria transmission, thereby laying the groundwork for the compartmental models used in epidemiology today.

In this talk, we develop discrete models of Tuberculosis (TB). This includes SEI endogenous and exogenous models without treatment. These models are then extended to a SEIT model with treatment. We develop two types of net reproduction numbers, one is the traditional $\mathcal{R}_0$ which is based on the disease-free equilibrium, and a new net reproduction number $\mathcal{R}_0(\mathcal{E}^*)$ based on the endemic equilibrium. It is shown that the disease-free equilibrium is globally asymptotically stable if $\mathcal{R}_0 \leq 1$ and unstable if $\mathcal{R}_0 > 1$. Moreover, the endemic equilibrium is locally asymptotically stable if $\mathcal{R}_0(\mathcal{E}^*) < 1 < \mathcal{R}_0$.

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Border collision bifurcations associated with a piecewise linear map degenerate period adding structure

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Abstract

Managing chaotic systems remains a crucial research area across various practical disciplines. A simple yet effective method for chaos control, used in both economics ([5]) and population biology ([3]), involves employing upper and lower limiters. These limiters activate when the state variable significantly rises or falls compared to the previous period, aiming to reduce the amplitude of fluctuations over time. Models incorporating these upper and lower limiters can be described by one-dimensional bimodal piecewise maps with a degenerate period adding bifurcation structure, as detailed in [1- 2] and [4]. Understanding this bifurcation structure is crucial for comprehending the map's long-term behavior and system dynamics. In this work, we demonstrate that the degeneracy not only influences the type of border collision bifurcations forming the bifurcation structure in the parameter space but also explains the abrupt transitions between different attractors. Our analysis elucidates how these bifurcations impact the system's stability and fluctuation patterns, providing deeper insights into chaos control mechanisms. Specifically, we present theoretical results and numerical investigations that offer a detailed description of the degenerate period-adding structure and a comprehensive characterization of the chaotic attractors' shapes. This extends the partial results previously reported by [1- 2] and [4].

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Global Bifurcation Analysis of Continuous and Discrete Dynamical Systems

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Abstract

We carry out a global bifurcation analysis of continuous and discrete dynamical systems [1]. First, using new bifurcation and topological methods, we solve *Hilbert's Sixteenth Problem* on the maximum number of limit cycles and their distribution for the 2D Leslie–Gower population dynamics system [2], reduced quartic Topp system [3], and Euler–Lagrange–Liénard polynomial system [4]. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for Lorenz-type systems connecting globally the Andronov, Shilnikov, homoclinic, period-doubling, period-halving and other bifurcations of limit cycles which is related to *Smale's Fourteenth Problem* [5]. We discuss also how to apply our approach for studying global limit cycle bifurcations of multi-parameter discrete dynamical systems which model the population dynamics in biomedical and ecological systems.

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Non-hyperbolic Bifurcation and Control of a Predator-prey Interaction with Harvesting and Predator Crowding

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Abstract

In general, a discrete-type dynamical system exhibits richer dynamical phenomena than that of its continuous counterpart. Keeping this in mind, a predator-prey system is considered with two species [5]. The prey species follows the logistic law of growth with a limited carrying capacity. The predator's response follows the Holling type-II functional response [2, 3, 4]. The prey and predator populations are harvested at a rate proportional to their population densities. The predator population has a natural death rate. There is a crowding effect of the predator population, the effect of which decreases the growth rate of the predator population. With the above biological phenomena, the predator-prey system is formulated with harvesting and the effect of crowding on the predator population. The system is formulated as continuous-time flow. It is then converted into the discrete-time map following [7]. The map is now analyzed for several qualitative properties starting from the steady state of the population to the stability behavior, bifurcation and the control of the map. A set of conditions have been derived for the existence of the fixed points. Two fixed points are found. One axial fixed point is obtained in which the predator population becomes extinct from the system. Another interior equilibrium point is obtained in which both the species: the prey and the predator population are in their positive level. The existence of the coexisting equilibrium point demands some threshold conditions of the system parameters. The stability property is analyzed using variational methods for the map in the neighborhood of the fixed points [5]. The criteria for the non-hyperbolic fixed points are derived. Using the Centre manifold theorem and the normal form method, the system is analyzed [1]. Under certain parametric restrictions, the map is analyzed for bifurcation leading to different types of orbits of the map. At the end of the study, a control parameter is used to make the model system a control system. The control system is made a feedback control system by feeding the state to the system. The feedback control system is then analyzed to control some dynamical behavior of the map [6]. Some numerical solution is obtained for supplementing the analytical results.

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Applications of Fuzzy Calculus to Dynamic Equations on Time Scales

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Abstract

During the talk, the applications of the innovative concepts (presented in the previous talk) of differentiability and integrability for fuzzy functions across arbitrary time scales will be presented. These concepts are founded on the principles of relative distance measure fuzzy arithmetic and horizontal membership functions. Utilizing these new concepts, results regarding the existence and uniqueness of solutions for first-order fuzzy dynamic equations on time scales will be derived. Furthermore, we investigate the continuity of solutions to fuzzy dynamic equations concerning variations in initial values. To illustrate the practicality of our findings, we present models in economics and biology that pertain to hybrid domains.

On the nonexistence of iterative roots of functions

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Abstract

An *iterative root* of order $n \geq 2$ of a self-map f on a nonempty set X is a self-map g on X such that $g^n = f$. We discuss a new result on the nonexistence of iterative roots of self-maps on arbitrary sets and use it to prove that every nonempty open set of the space $C([0,1]^m)$ of all continuous self-maps on the unit cube $[0,1]^m$ in $\mathbb{R}^m$ contains a map that does not have even discontinuous iterative roots of order $n \geq 2$. This, in particular, proves that continuous self-maps on $[0,1]^m$ with no continuous iterative roots at all are dense in $C([0,1]^m)$. The talk is based on our recent works [1,2].

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Analysis of a delay difference equation

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Abstract

In this presentation, we investigate the existence and uniqueness of solutions for a delay difference equation that represents the discrete Nicholsons blowflies type system. We utilize several fixed point theorems for this purpose. Furthermore, the Ulam-Hyers stability is explored.

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Dynamical behavior of a temporally discrete non-local reaction-diffusion equation on bounded domain

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Abstract

This talk focuses on the study of global dynamics of a class of temporally discrete nonlocal reaction-diffusion equations on bounded domains. Similar to classical reaction-diffusion equations and integro-difference equations, these equations can also be used to describe the dispersal phenomena in population dynamics. We first derived a temporally discrete reactiondiffusion equation model with time delay and nonlocal effects to model the evolution of a single species population with age-structured located in a bounded domain. By establishing a new maximum principle and applying the monotone iteration method, the global stabilities of the trivial solution and the positive steady state solution are obtained respectively under some appropriate assumptions.

Dynamics of a higher-order system of difference equations linked to generalized Balancing numbers

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Abstract

We propose theoretical interpretations regarding the depiction of solutions for to the following system of higher-order difference equations,

$$x_{n+1} = \frac{1}{A - y_{n-k}}, \quad y_{n+1} = \frac{1}{A - x_{n-k}}, \quad n, k \in \mathbb{N}_0.$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, and the initial conditions $x_{-k}, x_{-k+1}, \dots, x_0, y_{-k}, y_{-k+1}, \dots, y_0$ are non zero real numbers, wherein the solution correlates with generalized Balancing numbers. Furthermore, we explore the stability attributes and asymptotic tendencies of said equation.

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On the application of equations with piecewise constant arguments in stability of difference equations and numerical approximation of functional differential equations

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Abstract

In this talk first we recall our earlier works on obtaining stability results for difference equations with the help of differential equations with piecewise constant arguments (EPCAs). Then we we investigate numerical approximation of several classes of functional differential equations with the help of EPCAs. After summarizing some earlier works, a uniform approximation of a nonautonomous delayed Hopfieldtype impulsive neural network system is studied with an associated impulsive EPCA. Sufficient conditions are formulated, which imply that the error estimate of the numerical approximation decays exponentially with time on the half-line $(0, \infty)$.

A delay mathematical model for the dynamics of stem cells in leukemia with treatment

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Abstract

In this work, we investigate a time-delayed model describing the dynamics of the chronic myeloid leukemia model under treatment. First, we analyze the asymptotic behavior of the model. Next, a necessary and sufficient conditions is given for global stability of the trivial steady state. Moreover, the uniform persistence is obtained in the case of instability. Finally, we give some numerical simulation to illustrate our theoretical results.

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Discrete model for mosquito population replacement in heterogeneous environments

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Abstract

Mosquito-borne diseases kill more than 700,000 people each year around the world. A novel strategy to control such diseases employs Wolbachia, a maternally inherited endosymbiotic bacterium whose infection in mosquitoes may inhibit reproduction of viruses such as dengue and Zika, as well as malaria parasites. Thus releasing infected mosquitoes to invade wild mosquitoes is an effective measure of controlling mosquito-borne diseases. In this work, we divide the target area into exposed and non-exposed areas and consider different Wolbachia maternal heritability rates in the two areas. We assume that mosquitoes can migrate smoothly between two areas and analyze the effects of additional release and spraying of insecticides on maintaining high infection rates.

Angular Spectrum: A New Concept For Measuring Rotational Dynamics

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Abstract

We consider nonautonomous linear dynamical systems in discrete time

$$u_{n+1} = A_n u_n, \quad n \in \mathbb{N}_0, \quad u_n \in \mathbb{R}^d, \quad A_n \in \mathbb{R}^{d,d}, \quad \Phi \text{ solution operator.}$$

Angular values measure longtime averages of rotations of s-dimensional subspaces $V \subset \mathbb{R}^d$, i.e.

$$\alpha_n(V) = \frac{1}{n} \sum_{j=1}^{n} \measuredangle(\Phi(j-1,0)V, \Phi(j,0)V).$$

Here, the symbol $\measuredangle(\cdot,\cdot)$ denotes the maximal principal angle between two subspaces. Based on this notion, we introduce an angular spectrum that compromises all accumulation points for $\alpha_n(V)$ over all s-dimensional subspaces of $\mathbb{R}^d$. We show that only certain subspaces w.r.t. the dichotomy spectrum have to be considered. This reduction to trace spaces is particularly useful for numerical computations. For this task, we introduce a finite time version of the angular spectrum and discuss upper and lower semicontinuity. This leads to an algorithm that we apply to the famous Lorenz system.

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Spectrum invariance dilemma for nonuniformly kinematically similar systems

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Abstract

S. Siegmund [2] proved that if two systems are kinematically similar, then they have the same exponential dichotomy spectrum. In this talk, we unveil instances where nonautonomous linear systems manifest distinct nonuniform dichotomy spectra [1, 3, 4] despite admitting nonuniform kinematic similarity. In order to explore the theoretical foundations of this lack of invariance, we discern the pivotal influence of the parameters involved in the definition of nonuniform dichotomy.

Although spectra invariance might not be achieved, we present a weaker notion which can be verified and depends on nontrivial neighborhoods of the nonuniform dichotomy spectrum.

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Bifurcations and Arnold tongues of a multiplier-accelerator model

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Abstract

In this paper, we investigate the bifurcations of a multiplier-acceler-ator model with nonlinear investment function in an anti-cyclical fiscal policy rule. Firstly, we give the conditions that the model produces supercritical flip bifurcation and subcritical one respectively. Secondly, we prove that the model undergoes a generalized flip bifurcation and present a parameter region such that the model possesses two 2-periodic orbits. Thirdly, it is proved that the model undergoes supercritical

Neimark-Sacker bifurcation and produces an attracting invariant circle surrounding a fixed point. Fourthly, we present the Arnold tongues such that the model has periodic orbits on the invariant circle produced from the Neimark-Sacker bifurcation. Finally, to verify the correctness of our results, we numerically simulate an attracting 2-periodic orbit, a stable invariant circle, an Arnold tongue with rotation number 1/7 and an attracting 7-periodic orbit on the invariant circle.

Estimating critical intensities for bull and bear states

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Abstract

For a financial market with heterogenous agents, the stochastic sensitivity of coexisting market equilibria has been studied in [1]. In particular, relying on the concept of the *critical noise intensity*, i.e. the largest noise variance for which transitions are still unlikely events, we devise theoretical analytical measures of stress resistance of bull and bear states. In the current project, we explore ways to provide empirical estimates of these measures. Since the critical intensities depend in a complex manner on all parameters of linear map with 5 segments augmented by noise, their estimation requires precise estimates of all model parameters. Among the available estimation strategies, we focus on techniques devised for segmented regression models with unknown break-points [2], on Likelihood based estimation techniques devised for Self-Exciting Threshold Auto Regressive (SETAR) models [4], and on Monte-Carlo techniques (MCMC) for parameter estimation [3]. The operational characteristics of the estimators are demonstrated on the basis of artificial as well as on real-world capital market data.

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Stochasticity in Recruitment Ability on Collective Foraging Dynamics

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Abstract

This study introduces a two-dimensional collective foraging model and its stochastic counterpart, simplifying the previous three-dimensional framework for studying foraging activities in social insect colonies. Our analysis highlights the resemblance between the two-dimensional model and the classical three-dimensional framework. Subsequently, a comprehensive analysis of the long-term collective foraging dynamics in a stochastic environment is conducted, along with an investigation of the relationship between stochasticity and the transitions among different stable states of collective foraging dynamics.

Optimal control of a discrete-time plant–herbivore/pest model with bistability in fluctuating environments

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Abstract

Discrete-time plant-pest models with two different constant control strategies (i.e., removal versus reduction strategies) have been investigated to understand how to regulate the population of pest. The corresponding optimal control problem has been explored on three scenarios of bistability plant-pest dynamics where these dynamics are determined by the growth rate of the plant and the damage rate inflicted by pest. Furthermore, the impacts of fluctuating environments on discrete-time plant-pest dynamics have been explored. Through analysis and simulations, we identify and evaluate the optimal controls and their impact on the plant-pest dynamics. There are critical factors to characterize the optimal controls and the corresponding plant-pest dynamics such as the control upper bound (the effectiveness level of the implementation of control measures) and the initial conditions of the plant and pest. The results show that the pest is hard to be eliminated when the control upper bound is not large enough or the initial conditions are chosen from the inner point of the basin of attractions. However, as the control upper bound is increased or the initial conditions are chosen from near the boundary of the basin of attractions, then the pest can be manageable regardless of fluctuating environments.

The onset of Shilnikov chaos in a three-cell population model of cancer

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Abstract

Modeling the growth of cancer cells is one of the most important tasks in the field of studying living systems. This is an important tool in cancer research and the development of new treatments, allowing us to understand the mechanisms of interaction of infected cells with surrounding tissue and evaluate the influence of various external and internal factors on their growth. We study cancer growth dynamics in the framework of the de Pillis and Radunskaya model which describes the interaction between cancer cells and two types of effector cells [1-2]. However, we write it in the dimensionless Itic-Banks form [3]:

$$\begin{cases} \dot{x}_1 = x_1(1-x_1) - a_{12}x_1x_2 - a_{13}x_1x_3, \\ \dot{x}_2 = r_2x_2(1-x_2) - a_{21}x_1x_2, \\ \dot{x}_3 = \frac{r_3x_1x_3}{x_1+k_3} - a_{31}x_1x_3 - d_3x_3 \end{cases} \tag{9}$$

From the works [3-4] it is known that spiral chaos associated with the emergence of a Shilnikov saddle-focus loop can arise in system (9). We show that strange attractors are born as a result of the implementation of the Shilnikov scenario [5]. The main part of the work is devoted to the study of bifurcations of codimension two, which are organizational centers in the system under consideration. In particular, we describe bifurcation scenario for equilibrium state in the case where it has a pair of zero eigenvalues (Bogdanov-Takens bifurcation), as well as zero and a pair of purely imaginary eigenvalues (zero-AH bifurcation). It is shown how these bifurcations are related to the emergence of Shilnikov attractors.

This work is an output of a research project implemented as part of the Basic Research Program at the National Research University Higher School of Economics (HSE University).

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On new types of wild Lorenz-like attractors

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Abstract

In 1998 Turaev and Shiknikov introduce a new class of chaotic attractors – the so-called pseudohyperbolic attractors. The common feature of such attractors is the existence of a continuous splitting of the tangent space in a neighborhood of an attractor into a direct sum of two invariant linear subspaces E1 and E2: the linearized system restricted to E2 is uniformly contracting, whereas in E1 it uniformly expands volumes. Any possible contraction in E1 should be uniformly weaker than any contraction in E2. These garantee both the positivity of the maximal Lyapunov exponent and the preservation of this property at small perturbations of the system.

To the moment it is known several (but not too much) examples of pseudohyperbolic attractors in both system with continuous and discrete time. Lorenz and discrete Lorenz attractors are one of the well-known examples of such attractors. The Lorenz attractor is an attractor satisfying conditions of the Afraimovich-Bykov-Shilnikov geometrical model, it contains one saddle equilibrium with its unstable manifold. The wild Lorenz attractor is a discrete analogue of the Lorenz attractor. It has similar "shape" but, unlike the Lorenz attractor, contains orbits of homoclinic and heteroclinic tangencies. Note that only examples of the so-called "flow-like" discrete Lorenz attractors possessing close to zero second Lyapunov exponent are known to the moment. In this talk we will present pseudohyperbolic attractors of two new types: the Lorenz-like attractor containing a pair of saddle equilibria [1] and a "genuely discrete" Lorenz attractor possessing negative second Lyapunov exponent [2].

This work is an output of a research project implemented as part of the Basic Research Program at the National Research University Higher School of Economics (HSE University).

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Using stage structure to infer process-based models for species on the move

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Abstract

As climate change alters Earth's environment, understanding the mechanisms, pace, and dynamics of shifting species is of interest both ecologically and for conservation efforts. Predictions about how species will respond to environmental change are often made using models that neglect spatial population dynamics and that assume species are in equilibrium with their environment. In this talk, I will present stage-structured, process-based models for species distribution and abundance under the influence of climate change or variability and an approach for obtaining the parameters of such complex systems. Moreover, I will present simulation results obtained from applying one of the models and framework to Atlantic cod (Gadus morhua) data from Northeast U.S. region.

Boltzmann type-Machine in Darwinian Evolution Population Dynamics

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Abstract

Using information theory, we propose an estimation method for traits parameters in a Darwinian evolution model for species with on trait or multiple traits. We first use the Fisher's information to obtain the errors on the estimation for one species with one or multiple traits. We then estimate parameters by minimizing the relative information in a Darwinian evolution population model using either a classical gradient ascent or a stochastic gradient ascent. The proposed procedure is shown to be possible in a supervised or unsupervised learning environment, similarly to Boltzmann machines.

Analysis of a conformable fractional derivative system

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Abstract

In this work, a system of conformable fractional order derivative is considered, it is obtained from an immunogenic tumor model. First, to generate a discrete version of the conformable system, a discretization process is used. After that, the conformable derivative is taken into account when studying the stability of discrete equilibrium points. Finally, numerical simulations are used to study the dynamical behaviours of the system and to prove cases of chaos.

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Evolutionary adoption of behavioral rules with finitely many players

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Abstract

In this paper, we consider an evolutionary market model with a finite number of agents in discrete time. These agents have at their disposal a finite set of behavioral rules in order to forecast future prices and outputs to be placed in the market. For each behavioral rule, to which an implementation cost can be attached, we associate a fitness measure that is used to explain the evolutionary dynamics of the adoption of the prediction rules and market variables. In contrast to what is often assumed in the literature, the evolutionary modeling of the game is carried out on a finite population by means of a stochastic Moran process. This Moran process is considered under a time-varying fitness landscape under the influence of agent interaction. We present a characterization to obtain uniqueness of the equilibrium price and invariant distribution of the process. We then show an example with the adoption of two prediction rules, the rational and the naive ones, and show the possible dynamics of convergence and loss of stability that can be related to the considered Moran process.

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Solution space characterisation of perturbed stochastic Volterra integrodifferential equations

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Abstract

In this talk we present recent results which provide a characterisation of the solution space of perturbed linear Volterra integrodifferential equations and highlight extensions to stochastic equations with additive noise. Various function spaces are explored while particular effort is dedicated to when solutions lie in Lp spaces, in this case we illustrate links to the study of stochastic Volterra difference equations. We improve upon classical results by providing necessary and sufficient conditions on the problem data which ensure various types of stability, to this end we remove hypotheses which require pointwise estimates on the perturbation functions. This allows us to enlarge the known class of perturbation functions which preserve stability. Additionally we discuss how such results can also be used to study functional equations with additive noise and the mean square of both Volterra and functional equations with multiplicative noise. This is joint work with John Appleby.

On limit sets of Brouwer homeomorphisms generating foliations of the plane

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Abstract

We study properties of a Brouwer homeomorphism f for which there exists a foliation of the plane with the leaves being invariant lines of f. For such a homeomorphism f, we describe the form of the positive and negative limit sets of the discrete dynamical system defined by f. This form turns out to be similar to the form of the positive and negative first prolongational limit sets of continuous dynamical systems.

A flat control law for two diffusively y-coupled Rössler systems

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Abstract

Controlling dynamical systems and particularly, network, is of a primary interest, when the system (network) is of (nodal) high-dimensionality. Such a problem is intrinsically related to the analysis of the observability of the corresponding state space and its dual, the controllability of the system (network). An additional constrain can be added in requiring the possibility of rending the system flat. Starting from the placement of sensors providing a global observability, we address the dual problem of placing the actuators allowing to design a flat input [1,2,3]. Since global observability of network can be solved, when the nodes are y-coupled Rössler systems, by pairing the nodes, a first step in controlling network appears to design a flat control law for a pair of diffusively y-coupled Rössler systems. It is solved by inserting a differential delay.

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Some new quadratic conditions for Hamiltonian elliptic systems

Fangfang Liao

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Abstract

In this talk, we will introduce some recent results about Hamiltonian elliptic systems. More precisely, we propose a new asymptotically quadratic condition which is more general than usual condition, and we take advantage of the non-classical variational approach for strongly indefinite problem to establish some new existence results of nontrivial solution to Hamiltonian elliptic systems.The strongly indefinite features together with new asymptotically quadratic condition bring some new difficulties in our analysis. The results included in this talk complement several recent contributions to the study of Hamiltonian elliptic system.

Analysis of periodically spraying pesticides to control Nilaparvata lugens

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Abstract

Brown planthopper *Nilaparvata lugens* is a serious damaging rice pest that can transmit rice ragged stunt virus. The effective method to control *N.lugens* is to spray pesticide. The most crucial concern in designing spray strategies is how often pesticides should be sprayed in order to guarantee effective pest control. Based on hormetic dose-response model, we formulate and analyze a chemical control *N.lugens* model considering the situations for the spraying period T more than, or equal, or less than the duration of pesticide effect $\bar{T}$. The results show that system can exhibit threshold dynamic properties under the three spraying strategies, that is, under a certain threshold condition, the origin is globally asymptotically stable, and in the opposite cases of this condition, system appears a unique globally asymptotically stable positive period solution. In addition, numerical simulations show that shortening the period of pesticides spraying can either reduce or increase the number of wild *N.lugens* for the case of either $T > \bar{T}$ or $T < \bar{T}$. By comparing three spray strategies, the results show that strategy of spraying less often is better than other spray methods and can effectively control wild *N.lugens* in the shortest time.

Discrete Models in Econophysics: Inequality in Random Markets

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Abstract

Some economic discrete models for random conservative markets are addressed in this communication. In these models the agents trade by pairs bringing the system toward an statistical equilibrium, this is the asymptotic wealth distribution. The time evolution of these models are given by nonlinear functional mappings. These maps are nonlinear operators in the space of wealth distributions, which are shown to conserve the total and mean wealth of the economic system. Different asymptotic results for several models are presented. The decay to the exponential distribution or gamma-like distributions are found in some of these models. Simulations and implementations of these systems in different topologies and in different situations are also presented. As a common point of all these models, inequality is reached in the collectivity as a natural consequence of randomness without the need of some special force or intervention from the exterior. Even more, different types of randomness give place to different kinds of natural inequality. The distribution in economic classes for the different models are shown.

Positive solutions for a system of fractional q-difference equations with multi-point boundary conditions

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Abstract

We study the existence, uniqueness, and multiplicity of positive solutions to a system of fractional q-difference equations that includes fractional q-integrals. This investigation is carried out under coupled multi-point boundary conditions featuring q-derivatives and fractional q-derivatives of various orders. The proofs of our principal findings employ a range of fixed point theorems, including the Guo-Krasnosel'skii fixed point theorem, the Leggett-Williams fixed point theorem, the Schauder fixed point theorem, and the Banach contraction mapping principle.

How far can we go in stability conditions for periodic maps?

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Abstract

In this talk we revisit the necessary and sufficient conditions for asymptotic stability of periodic cycles for periodic difference equations by using the Jury's conditions. Such conditions are obtained using the information of the Jacobian matrices of the individual mappings, avoiding thus the computation of the Jacobian matrix of the composition operator, which in higher dimension can be an a very difficult task. We illustrate our ideas by using models in population dynamics and in economics game theory.

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Exponential stability for discrete dynamics via evolution maps

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Abstract

For a discrete dynamical system defined by a sequence of bounded linear operators, we present a complete characterization of exponential stability in terms of invertibility of a certain operator associated to the evolution map of the dynamics. The invertibility of this operator is connected to the existence of a particular type of admissible exponents. Some adequate examples are exposed to emphasize some significant qualitative differences between uniform and nonuniform behavior.

Multidimensional discrete generating series of solutions to difference equations

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Abstract

Let $\mathbb{Z}_\geqslant$ denote the nonnegative integers, $\mathbb{Z}^n = \mathbb{Z} \times \cdots \times \mathbb{Z}$ be the n-dimensional integers, $\mathbb{Z}^n_\geqslant = \mathbb{Z}_\geqslant \times \cdots \times \mathbb{Z}_\geqslant$ for $n \in \mathbb{Z}_\geqslant$ be its nonnegative orthant. For any $z \in \mathbb{C}$ and $n \in \mathbb{Z}_\geqslant$, we define $z^{\underline{n}} = z(z-1)\cdots(z-n+1)$. We will use the multidimensional notation as $x = (x_1, \ldots, x_n) \in \mathbb{Z}^n_\geqslant$, $z = (z_1, \ldots, z_n) \in \mathbb{C}^n$, $\xi = (\xi_1, \ldots, \xi_n) \in \mathbb{C}^n$, $\xi^x = \xi_1^{x_1} \cdots \xi_n^{x_n}$, $z^{\underline{x}} = z_1^{\underline{x_1}} \cdots z_n^{\underline{x_n}}$, $\ell = (\ell_1, \ldots, \ell_n) \in \mathbb{Z}^n_\geqslant$, $x! = x_1! \ldots x_n!$. We also will use $x \leqslant y$ for $x, y \in \mathbb{Z}^n$ componentwise.

For $f\colon \mathbb{Z}^n_\geqslant \to \mathbb{C}$, we define multidimensional discrete generating series of f as $F(\xi;\ell;z) = \sum\limits_{x \in \mathbb{Z}^n_\geqslant} f(x)\xi^x z^{\underline{\ell x}}$. For polynomials $p_\alpha \in \mathbb{C}[z]$ and finite set $A \subset \mathbb{Z}^n_\geqslant$ we denote the difference equation $\sum\limits_{\alpha \in A} p_\alpha(x) f(x-\alpha) = 0$. and its initial data on a set $X_m = \mathbb{Z}^n_\geqslant \setminus \left(m + \mathbb{Z}^n_\geqslant\right)$ and we define the initial data function $\varphi : X_m \to \mathbb{C}$ so that $f(x) = \varphi(x), \in X_m$.

For $\delta_j : x \to x + e^j, j = 1, \ldots, n$ we define the polynomial difference operator $P(\delta) = \sum\limits_{0 \leqslant \alpha \leqslant m} c_\alpha \delta^\alpha$. We introduce the shift operator and its truncation by $\mathcal{P}(\xi;\ell;z) = \sum\limits_{0 \leqslant \alpha \leqslant m} c_\alpha \xi^\alpha z^{\underline{\ell\alpha}} \rho^{\ell\alpha}$ and $\mathcal{P}_\tau(\xi;\ell;z) = \sum\limits_{\substack{0 \leqslant \alpha \leqslant m \\ \alpha \ngeqslant \tau}} c_\alpha \xi^\alpha z^{\underline{\ell\alpha}} \rho^{\ell\alpha}$, and the discrete generating series of the initial data for $\tau \in X_m$ by $\Phi_\tau(\xi;\ell;z) = \sum\limits_{x \ngeqslant \tau} \varphi(x)\xi^x z^{\underline{\ell x}}$.

For $\Delta_j F(z) = F(z+e^j) - F(z), j = 1, \ldots, n$, $\rho_j F(z) = F(z - e^j)$ and $\theta_j = \ell_j^{-1} z_j \rho_j \Delta_j$ we define an operator $\mathcal{P}_A(\xi;\ell;z;\theta;\rho) = \sum\limits_{\alpha \in A} p_\alpha(\theta+\alpha)\xi^\alpha z^{\underline{\ell\alpha}} \rho^{\ell\alpha}$.

In this research functional equations for the product $\mathcal{P}(\xi;\ell;z)F(\xi;\ell;z)$ and $\mathcal{P}_A(\xi;\ell;z;\theta;\rho)F(\xi;\ell;z)$ are considered and their propertires is studied.

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Convergent solutions to non-autonomous difference equations

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Abstract

Sufficient conditions are given for the convergence of solutions to non-autonomous systems of difference equations in $\mathbb{R}^n$ to the equilibrium of the associated autonomous system. We are focused on Kolmogorov maps. Several examples, that include n-dimensional Leslie-Gower system and n-dimensional Ricker system illustrate the results.

A generalization of the Maxwell integral model: the Maxwell-Mittag-Leffler model

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Abstract

In this work, we investigate a time-delayed model describing the dynamics of the chronic myeloid leukemia model under treatment. First, we analyze the asymptotic behavior of the model. Next, a necessary and sufficient conditions is given for global stability of the trivial steady state. Moreover, the uniform persistence is obtained in the case of instability. Finally, we give some numerical simulation to illustrate our theoretical results.

On chaos for inducing monotone regular curve maps on hyperspaces

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Abstract

I will present new results concerning transitivity, nonwandering sets and chaos for inducing monotone regular curve maps on hyperspaces. If X is a regular curve and $f : X \to X$ be a monotone map, denote by $2^f : 2^X \to 2^X; F \mapsto f(F)$ the induced map on the hyperspace 2^X. The dynamical properties of the 2^f are investigated. If $P(f)$ (resp. $P(2^f)$) is the set of periodic points of f (resp. 2^f), we show that $P(f)$ is dense in X if and only if $P(2^f)$ is dense in 2^X. Moreover, we show that 2^f has zero entropy provided that f is nonwandering. These results generalize those proven in (1), (2), (3), for monotone dendrite maps, graph maps, homeomorphism regular curve maps, respectively.

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Asymptotic constancy and asymptotic solutions of difference equations in a Banach space

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Abstract

This study is concerned with the difference equation $x(n+1) = Tx(n) + y(n)$, where T is a bounded linear operator acting in a Banach space, and y is assumed to be bounded. We focus on asymptotically constant and asymptotic solutions of this equation under the assumptions that the point 1 is not in the spectrum of T on the unit circle or is its isolated element. Some examples are provided to illustrate the obtained results. This is a joint work with Nguyen Van Minh, Nguyen Duc Huy, and Vu Trong Luong.

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Homoclinic solutions for a class of partial difference equations

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Abstract

In this talk, we consider a class of partial difference equations with sign-changing mixed nonlinearities. Using critical point theory, we obtain the existence results of homoclinic solutions in the periodic case. In the case of unbounded potential functions, we derive the existence and multiplicity of homoclinic solutions, including the scenarios of at least one solution, two solutions, two positive solutions, and infinitely many solutions. Our conditions allow for the non-existence of limits of f/u both at the origin and at infinity, which of course means that our conditions encompass cases of superlinear, asymptotically linear and a mixture of them.

An effective approach for numerical solution of delay difference equations

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Abstract

A new technique that involves the application of the HAAR wavelet collocation method is introduced to find numerical solutions to linear and non-linear delay difference equations. The obtained solutions are compared with other existing solutions. Numerical examples are presented to show the robustness and reliability of the proposed method. The convergence results and error analysis are also discussed. It is demonstrated that the accuracy of the result increases as the degree of resolution increases.

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From discrete random walks to epidemic spreading in complex networks with mortality

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Abstract

Discrete random walks in complex random graphs offer a powerful tool to mimic the complexity of real world mobility patterns of individuals. We explore epidemic spreading of a vector transmitted disease in complex networks by a multiple random walker's approach. Each random walker performs an independent simple random walk in networks such as Barabasi-Albert (BA), Erdös-Rényi (ER) and Watts-Strogatz (WS) type graphs. We assume, both walkers and nodes can be infected. They are in one of the compartments, susceptible (S) or infected (I) representing their states of health. The transmission of the disease happens as follows. Susceptible nodes may be infected by visits of infected walkers, and susceptible walkers may be infected by visiting infected nodes. No direct transmission among walkers and among nodes is possible. In addition, for infected walkers, we account for the possibility that they may die during the random time span of their infection, whereas infected nodes never die and always recover after a finite random period of infection. This model mimics a large class of diseases such as Dengue and Malaria with transmission pathways via vectors (mosquitoes). Based on these assumptions

we develop a stochastic mean field model for the compartmental fractions with mortality and implement the assumptions of the model into random walk simulations. We compare the spreading in the mean field model and the random walk simulations and explore the effect of the topology of the network. From stability analysis of the healthy state with and without mortality we derive the respective basic reproduction numbers R_M, R_0 where $R_M, R_0 > 1$ is the condition that the disease is starting to spread. We show that $R_M < R_0$. It turns out that for zero mortality for $R_0 > 1$ a stable endemic equilibrium exists which we obtained in explicit form and which is independent of the initial conditions. The model has applications in a wide range of spreading phenomena beyond epidemic dynamics such as propagation of contaminants, wood fires, and the kinetics of certain chemical reactions. An animated simulation of the spreading on a WS graph can be seen by clicking here. For the details of our model we refer to [1-2].

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On Discrete Generalized Nabla Fractional Sums and Differences

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Abstract

This study is devoted to investigating a class of discrete nabla fractional differences and sums based on the convolution of two input functions. This together with the idea of discrete Riemann-Liouville and Caputo operators enable us to define the discrete generalized nabla fractional differences and sums in both senses. The concept of dual identities is used here to make a relationship between our class and the class of discrete delta fractional operators. This relationship is justified and clarified via two specific examples depicting discrete power formulas. Furthermore, via new classes of discrete nabla fractional operators containing exponential and Mittag–Leffler function, discrete generalized nabla Caputo-Fabrizio-like and Atangana-Baleanu-like fractional differences and sums are constructed for the system. Finally, a discrete version of the fundamental theorem of calculus is proved in our class of discrete nabla fractional operator space.

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Evolution of partial migration in stochastic environments

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Abstract

Partial migration is a unique form of phenotypic diversity wherein migrant and non-migrant individuals coexist in a population. It has been found across many taxa, including fish, invertebrates, and mammals. In general, partial migration population models with deterministic density dependence effects are studied, and important biological features like environmental stochasticity have been neglected. Predicting the impacts of climate change on biological populations is likely to become one of the significant scientific challenges of this century. Also, the stochasticity in the environment can play an essential role in escaping the population from extinction. So, the goal is to investigate the impact of environmental fluctuations on the partial migration population. In this talk, we discuss two stochastic models, one incorporating stochasticity in fertility function and the other accessing the environmental disturbances occurring at random times. We discuss the stochastic persistence, extinction, and predicting ecological impacts on migrating species. In the case of persistence, we discuss how environmental uncertainty affects the evolutionary stable strategies of the partial migration population.

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Time-fractional order models for charge transport in disordered semiconductor materials

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Abstract

This work reports on the numerical modelling of electrical currents through a thin layer of a disordered semiconductor material sandwiched between two parallel electrodes, under the influence of an externally applied electric field directed normally to the electrodes. One of the tools used with success to describe the charge transport in these kind of materials, e.g. organic semiconductors, is the multiple trapping model ([1]) which, under certain conditions, leads to anomalous diffusion equations that contain time fractional derivatives. Continuing the work initiated in [2], we will focus on the numerical approximation of the involved problems and, in particular, the cases where the charge carrier mobility is dependent on de carrier concentration [3], which leads to a nonlineal diffusion coeficient in the drift diffusion equations.

Acknowledgments This work is financially supported by national funds through the FCT/MCTES (PIDDAC), under the project 2022.06672.PTDC - iMAD - Improving the Modelling of Anomalous Diffusion and Viscoelasticity: solutions to industrial problems, with DOI 10.54499/2022.06672.PTDC
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Finite Difference schemes for distributed-order diffusion equations

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Abstract

We focus on the numerical analysis of distributed-order fractional differential equations, in which the fractional derivatives are given in the Caputo sense. Finite difference schemes on non-uniform meshes are presented and analysed in terms of stability and accuracy. We further explore mesh adaptive algorithms. Although such schemes based on *a posteriori* error analysis already exist for Caputo fractional differential equations ([1-2]), to the best of our knowledge, these can not be found in the literature for distributed-order equations. Some numerical experiments and results are provided.

Acknowledgments This work is financially supported by national funds through the FCT/MCTES (PIDDAC), under the project 2022.06672.PTDC - iMAD - Improving the Modelling of Anomalous Diffusion and Viscoelasticity: solutions to industrial problems, with DOI 10.54499/2022.06672.PTDC (https://doi.org/10.54499/2022.06672.PTDC)

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Hysterical Dynamics of player's behavior in a welfare coalition game

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Abstract

We introduce a coalition welfare game model with a group of homogeneous players who have the same characteristics. In this coalition model, the players' preferences are described by a class of utilities that are used to construct a quadratic welfare function. We derive three main welfare thresholds to distinguish the players' preferences when they adapt a certain behavior. We determine all optimal welfare strategies in this coalition game and we classify them. We use geometry to show the existence of one periodic cohesive hysterical dynamics in the player's behavior when the influence parameter is positive. Furthermore, we show that there are two hysterical dynamics in the player's behavior when the influence parameter is negative. However, if the influence parameter is zero, then all players may split in an infinite hysterical dynamics around the bifurcation threshold. Finally, we determine the maximum welfare occurrence based on the type of the welfare strategies.

Resonant torus doubling and hyperchaos in three-dimensional maps

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Abstract

Torus doubling bifurcations occur in maps of dimensions greater than or equal to three. Recently, it was shown that resonant torus can also undergo a doubling bifurcation similar to ergodic torus [1, 2, 3]. After subsequent doublings, formation of Shilnikov attractors and hyperchaotic attractors are usually observed [4]. It is shown that the 3D quadratic map under consideration shows both resonant and ergodic torus doubling bifurcation. We show that the system exhibits resonant torus-doubling bifurcation in which the length doubled mode-locked periodic orbits lies on a M¨obius strip. I will then discuss about the strong hyperchaoticity exhibited by the map. The map under consideration displays strong hyperchaoticity in the sense that in a wider range of parameter space the system showcase all three positive Lyapunov exponents. It is shown that the saddle periodic orbits eventually become repellers at this hyperchaotic regime. By computing the distance of the unstable periodic orbits to the attractors as a function of parameters, it is shown that the hyperchaotic attractors absorb the repelling periodic orbits. Finally, I will discuss various routes to this hyperchaotic regime and role played by the saddle periodic orbits.

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Periodic forcing of a 3D volume-preserving flow with a bubble of stability

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Abstract

We shall consider a 3D volume-preserving flow with two saddle-foci equilibria, whose 2D stable and unstable invariant manifolds coincide, creating a 2-sphere foliated by spiraling heteroclinic orbits. This sphere, usually referred to as a bubble of stability, is generically created at a Hopf-zero bifurcation. We will consider the effect of a non-autonomous periodic perturbation on the system. This perturbation introduces an additional frequency, which interacts with the intrinsic frequency related to the multiplier of the foci. In particular, our focus will be on the splitting of the 2D stable and unstable invariant manifolds and its quasi-periodic asymptotic behaviour. Such a description naturally lead to some new and open questions related to chaos formation in such systems.

A generalized Beddington host-parasitoid model with an arbitrary parasitism escape function

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Abstract

Our main goal is to analyze the asymptotic behavior of a generalized Beddington host-parasitoid model with an arbitrary parasitism escape function. From our study, we obtain the existence of three types of equilibrium points: extinction, exclusion and coexistence. We establish their local stability and for the extinction and exclusion equilibrium we are able to obtain global stability results. Furthermore, we analyze the possible occurrence of bifurcations in the model. Concretely, we show that the exclusion equilibrium undergoes period-doubling bifurcation with a stable two-cycle and a transcritical bifurcation, creating a threshold for parasitoids to invade. Also, for the coexistence equilibrium we analytically demonstrate a period doubling and Neimark-Sacker bifurcation. Moreover, we prove the permanence of the system within a specific parameter space. Finally, we developed numerical simulations for concrete parasitism escape functions in order to show our theoretical results.

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Hammerstein integrodifference equations - structural properties and their applications

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Abstract

We consider structural properties such as monotonicity or subhomogeneity of Hammerstein vector-valued integral operators over compact domains involved in nonautonomous integrodifference equations. We show consequence of these properties on the dynamics of Hammerstein integrodifference equations in a local-global stability principle.

Huygens synchronization of three clocks

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Abstract

We investigate the synchronization and stability of mutually coupled oscillators through small impacts and explore both master-master and master-slave synchrony. The synchronization behaviour between two oscillators, one operating at a frequency ω and the other at a frequency near a multiple of the first one, $n\omega$ where n is an integer greater than 1, is investigated. It is observed that such systems tend to exhibit a consistent pattern of master-slave relationship. This phenomenon is geometrically elucidated by considering the interaction dynamics: while the multiple bursts of the faster oscillator tend to cancel each other out upon impacting the slower oscillator, the single burst of the slower oscillator on the faster has a secular effect. We propose that this synchronization pattern, arising from perturbative impacts, is prevalent across various physical systems. Further experimental validation of these findings could contribute to a deeper understanding of synchronization phenomena and their implications in natural and technological domains.

Ulam type stability for dynamic equations on a discrete time scale with two step sizes

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Abstract

This study is based on joint works with Professor D. R. Anderson (Concordia College, USA). In this talk, we deal with Ulam stability for first-order linear dynamic equations on a specific time scale with two alternating step sizes, where the coefficient is allowed to be complex valued. Ulam stability is a concept that guarantees that a true solution exists near any approximate solution. In this study, we show that the error between the approximate solution and the true solution is finite, and we explicitly find a constant called the Ulam constant that indicates the degree of the error. In addition, we also show that the Ulam constant is minimum.

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Classification of Nonoscillatory Solutions of a Third Order Nonlinear Dynamic Equation

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Abstract

Oscillation and nonoscillation theories play very important roles in gaining information about the long-time behavior of solutions of a dynamic equation. Therefore, we investigate the asymptotic behavior of nonoscillatory solutions as well as the existence of such solutions so that one can determine the limit behavior. For the existence, we use some fixed point theorems such as Schauder's fixed point theorem and the Knaster fixed point theorem. Specifically, we consider a third order nonlinear dynamic equation on time scales to investigate the nonoscillatory solutions . In addition to existence of solutions of such equation, we also consider the corresponding linear dynamical equation to show the some type of solutions cannot be eliminated and always exist on time scales.

Fractional diffusion on networks

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Abstract

We present an introduction to the fractional diffusion equation and the Lévy flights associated with the non-local diffusive transport. Motivated by this equation that is a generalization of the diffusion equation for the Brownian motion, we discuss its equivalent when the dynamics occur on networks; in particular, the concept of fractional Laplacian of a graph. For the fractional diffusion on networks, we present analytical expressions for the fractional transition matrix, the fractional degree, and the average probability of return of the random walker. Through all this work, we analyze the mechanisms behind fractional transport on networks and how long-range dynamics emerge through this approach.

Bifurcation Structures in a Discontinuous 2D Map, Modeling Exchange Rate Dynamics

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Abstract

In the present paper, we investigate the complex dynamics arising from a behavioral exchange rate discontinuous model with heterogeneous agents. Unlike previous works explaining the emergence of chaos in the exchange rate models as the resulting of nonlinearity, our model is able to produce endogenous exchange rate dynamics due to the presence of discontinuity induced by a sentiment index, which affects the way investors take their trading decisions.

We show that our model, represented by a two-dimensional discontinuous map, has the ability to produce interesting endogenous exchange rate dynamics. In particular, we study three different parts of the overall bifurcation structure in the parameter space. The first part is located in the neighborhood of an organizing center associated with a continuity breaking bifurcation. It is characterized by an infinite number of BCB curves issuing from a single point and the periodicity regions close to this point are ordered according to the period adding bifurcation structure. For the parameter values being sufficiently distant from this organizing center, there may exist other period adding structures, each being placed between two disjoint periodicity regions related to cycles having the same period but associated with symbolic sequences that differ by one letter. Finally, between the regions associated with period two, there may exist a particular patchwork-like bifurcation structure related to cycles of even periods. We have shown that some of periodicity regions belonging to this structure are organized according to the period adding, while ordering of the others correspond to period incrementing principle.

Nonlinear dynamics of a simple behavioral model of inflation

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Abstract

In this work we analyze the dynamics of a behavioral model of inflation with heterogeneous agents, along the lines of Cornea et al. (2019). Agents, albeit knowing

the economic fundamentals, and thus being able to derive a fundamental value of inflation, create beliefs about what the deviation of actual inflation from this fundamental path will be in the next period. In particular, we consider two versions of the model by including different types of linear expectations, namely mean reverting, trend extrapolating and anchoring and adjusting rules. Agents are also allowed to switch between these different forecasting strategies based on their recent relative forecasting performance.
We investigate the local stability of the fundamental steady state and analytically show the possible occurrence of a degenerate Neimark-Sacker bifurcation. Moreover, the global analysis of the model highlights the coexistence of different attractors associated with a subcritical Neimark-Sacker bifurcation which, in turn, takes place as the effect of an increase of the trend extrapolating forces.

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Abstract resolvent families on time scales

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Abstract

In this talk, we present a formulation of abstract resolvent family (also named resolvent operator), to describe formulas of solutions to dynamic equations on time scales [1], [2], of order $0 < \alpha \leq 1$. To this purpose, and based on the articles [3] and [4], we make use of a definition of Laplace Transform, to obtain a discrete counterpart of important properties in fractional calculus. In addition, we study the relationship between a resolvent family and its infinitesimal generator, properties of such families, and existence of solutions to abstract functional equations.

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Analysis of the 5-piece dynamical model of financial markets

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Abstract

We study a model of price dynamics in financial markets, taking into account the presence of two types of agents [1]. The model is defined by piecewise smooth map with two discontinuity points and two kink points.

In the deterministic case, we analyze the existence and stability of regular regimes and describe bifurcation scenarios. The parametric conditions for the border collision bifurcation, leading to the disappearance or appearance of cycles, are obtained. Due to the fact, that the map has both kinks and discontinuities, mixing of dynamic scenarios is observed (such as period adding and period incrementing).

In the case of the influence of a random disturbance, we use stochastic sensitivity function technique [2-3] and its associated method of confidence bands to describe noise-induced phenomena such that transitions between equilibria or cycles, intermittency, transitions between chaotic and equilibrium modes, generation of large-amplitude oscillations. We determine the critical intensity values that are necessary for the occurrence of stochastic phenomena.

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Nonstandard finite difference method for a microbial population model incorporating environmental stress

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Abstract

Microbial populations depend on their environment but can also modify it. In addition to breaking down complex nutrients for their growth, microbes can exhibit negative behavior by engineering the environment in ways that are detrimental to their proliferation. In this work, proposed is a deterministic mathematical model representing microbial populations responding to environmental changes such as toxins produced by the microbes accounting for the switch of cells to dormancy at high concentrations. A dynamically consistent nonstandard finite difference scheme is designed. Theoretical and numerical investigation of the proposed model is presented to provide insight into the conditions that may lead to microbial population extinction or oscillations.

Shadowing and Hyperbolicity for Linear Delay Difference Equations

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Abstract

This is a joint work with Professors Lucas Backes (Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil) and Davor Dragičević (University of Rijeka, Rijeka, Croatia).

It is known that hyperbolic linear delay difference equations have the positive shadowing property. In this talk, we show the converse and hence the equivalence between hyperbolicity and the positive shadowing property for the following two classes of linear delay difference equations: (a) for nonautonomous equations with finite delays and uniformly bounded compact coefficient operators in Banach spaces, (b) for Volterra difference equations with infinite delay in finite dimensional spaces.

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Bifurcation structures of a two-dimensional piecewise linear discontinuous map: Analysis of a cobweb model with regime-switching expectations

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Abstract

We consider the bifurcations occurring in a two-dimensional piecewise-linear discon tinuous map that describes the dynamics of a cobweb model in which firms rely on a regime switching expectation rule. In three different partitions of the phase plane, separated by two discontinuity lines, the map is defined by linear functions with the same Jacobian matrix, having two real eigenvalues, one of which is negative and one equal to 0. This leads to asymptotic dynamics that can belong to two or three critical lines. We show that when the basic fixed point is attracting, it may coexist with at most three attracting cycles. We have determined their existence regions, in the two-dimensional parameter plane, bounded by border collision bifurcation curves. At parameter values for which the basic fixed point is repelling, chaotic attractors may exist - either one that is symmetric with respect to the basic fixed point, or, if not symmetric, the symmetric one also exists. The homoclinic bifurcations of repelling cycles leading to the merging of chaotic attractors are commented by using the first return map on a suitable line. Moreover, four different kinds of homoclinic bifurcations of a saddle 2-cycle, leading to divergence of the generic trajectory, are determined.

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On dynamical behavior of discrete time non-linear higher-order fuzzy difference equation

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Abstract

In this presentation, we analyze the behavior of a nonlinear higher-order fuzzy difference equation. We explore the existence, positivity, and uniqueness of solutions to the aforementioned fuzzy difference equation. Moreover, we derive certain sufficient conditions regarding the qualitative dynamics, including boundedness, persistence, and convergence of positive fuzzy solutions of the model. Additionally, we provide two simulation examples to validate our theoretical analysis.

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Fractional diffusion on networks

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Abstract

We present an introduction to the fractional diffusion equation and the L´evy flights associated with the non-local diffusive transport. Motivated by this equation that is a generalization of the diffusion equation for the Brownian motion, we discuss its equivalent when the dynamics occur on networks; in particular, the concept of fractional Laplacian of a graph. For the fractional diffusion on networks, we present analytical expressions for the fractional transition matrix, the fractional degree, and the average probability of return of the random walker. Through all this work, we analyze the mechanisms behind fractional transport on networks and how long-range dynamics emerge through this approach.

Difference and Differential Representations of Quadratic Operator on $su(1,1)$

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Abstract

The present research outlines different representations of the spectral problem for the one-dimensional quadratic operator:

$$\hat{H} = \hat{Y}_1^2 - \hat{Y}_2^2 + \alpha(\hat{A} - \beta)^2,$$

where the operator $\hat{H}$ is considered on the irreducible Hermitian representation of the Lie algebra $su(1,1)$ and the Hermitian operators $\hat{A}$, $\hat{Y}_1$, $\hat{Y}_2$ are generators of this algebra. We consider the spectrum of the operator $\hat{H}$ in semiclassical approximation, in other words, when $\hbar \to 0$. In a talk, we are going to show that different representations provide deeper insight of the considered problem from the prospect of theory of differential and difference equations.

In [3], it has been shown that representation in $l_2(\hbar\mathbf{Z}_+)$ of the spectral problem for the operator $\hat{H}$ is the fourth-order homogeneous difference equation:

$$\frac{1}{2}a(x+2h)y(x+2h) + b(x)y(x) + \frac{1}{2}a(x)y(x-2h) = Ey(x),$$

$$a(x) = \sqrt{x(x+1)(x-h)(x+1-h)}, \quad b(x) = \alpha(x-\beta)^2.$$

The WKB method for difference equations is used to construct complex asymptotics of solutions [1]. However, construction asymptotics in a neighborhood of the singular point $x = 0$ is a difficult task. On the other hand, it is convenient to numerically construct eigenvalues and eigenfunctions of the operator via the difference equation.

Coherent states and an unitary coherent transform reduce the spectral problem to the analysis of the second-order differential operator in the space of holomorphic functions [2]. The corresponding spectral problem has a form of Schrodinger equation:

$$\hbar^2 u''(z) - \left(V(z) + Ea_0(z) + O\left(\hbar^2\right)\right) u(z) = 0,$$

$$V(z) = \frac{z^4\alpha(1+2\beta)^2 - 2z^2 + \alpha(1-2\beta)^2}{2\left(z^4 - 2\alpha z^2 + 1\right)^2}, \quad a_0(z) = -\frac{2}{z^4 - 2\alpha z^2 + 1}.$$

Asymptotic formulas for the tunnel splitting of the energies has been proved via the complex WKB method. Unlike the classical formula for one-dimensional Schrodinger operator with a double-well potential, the splitting formula implies the existence of the oscillating factor[4]. Note that solutions must be analytic near the singular points for $|z| < 1$. Hence, it is difficult to construct a numerical solution of the differential equation.

Moreover, we have constructed another representation of the spectral problem in Darboux coordinates in the space $L_2(\mathbf{R})$. It has a form of the fourth-order differential equation:

$$\frac{\alpha - 1}{4}\hbar^4 \tilde{y}^{(4)}(x) - \left(\alpha\left(\frac{1}{2}x^2 + \frac{1}{2} - \beta\right) - \frac{1}{2}\right)\hbar^2 \tilde{y}''(x) - \alpha\hbar^2 x\tilde{y}'(x) +$$
$$\left(+\alpha\left(\frac{1}{2}x^2 + \frac{1}{2} - \beta\right)^2 - \left(\frac{x^2}{2} + \frac{1}{2}\right)^2 - \frac{1}{4} + O(\hbar^2)\right)\tilde{y}(x) = E\tilde{y}(x).$$

From the differential equation, it immediately follows that the spectrum of the operator will be discrete if $\alpha > 1$. Moreover, the representation in $L_2(\mathbf{R})$ has the form of the fourth-order differential equation instead of the second-order as in case of the one-dimensional Schrodinger operator. It shows that the behaviour of the tunnel splitting of energies for the considered operator differs.

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Complex Behaviors of a Predator-Prey Model Discretized by Piecewise Constant Arguments

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Abstract

It is well known that two kinds of equations, differential and difference equations, are used in the modelling of population dynamics. When the size of the population is rarely small or there are no overlapping generations, using difference equations is more appropriate. Moreover, it is also known that difference equations have richer dynamical properties than continuous systems. In view of this observation, in this talk we consider a predator-prey model, and first of all, we discretize the model with the help of piecewise constant arguments. The best advantage of this technique is that it avoids the existence of negative solutions in the discretized system. This method creates a bridge between differential and difference systems. Then we start the investigation of the types of fixed points and analyze the flip and Neimark-Sacker bifurcations using bifurcation theory and center manifold theorems. We also aim to construct the conditions that make our system chaotic. We support the results obtained here with some numerical experiments.

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The Perron–Frobenius Theorem for Cubic Doubly Stochastic Matrices

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Abstract

The classical Perron-Frobenius theorem for a *positive* (*primitive*) square stochastic matrix plays an important role in addressing the consensus problem within multi-agent systems governed by *linear* protocols. According to this theorem, a linear stochastic operator associated with the *positive* (*primitive*) square stochastic matrix possesses a unique stationary distribution (fixed point) within the simplex. Furthermore, its trajectory (orbit), starting from any point within the simplex, always converges to this unique stationary distribution (fixed point). An important question arises regarding the feasibility of generalizing *the classical Perron-Frobenius theorem*, extending from *positive* (*primitive*) *square* stochastic matrices to *positive* (*diagonally primitive*) *cubic* stochastic matrices. The result of this nature is significantly important in the *nonlinear* context of consensus problems within multi-agent systems. In the context of *cubic* stochastic matrices, the situation is more complex than one might anticipate. In this regard, some supporting examples will be provided in order to support our standpoint. The primary goal of this talk is to extend *the classical Perron-Frobenius theorem* from *square doubly stochastic matrices* to *cubic doubly stochastic matrices*. Additionally, we also discuss a related *global stability problem* in the context of both *autonomous* and *non-autonomous* dynamical systems. This talk is based on the results published in the papers [1,2,3].

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Dynamics of Nash strategic maps in a simple game model with discrete preference

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Abstract

We introduce a simple game with one group of players who have same interest. We model their behavior according to discrete utility which is based on two factors: The first factor is the self-independent taste effect and the second factor is the crowding dependent type effect. We assume that all players can decide one of two alternative decisions and the distribution of the player's decision is described by a pure or mixed strategic map. We classify all strategic maps and characterize all the ones that form Nash equilibria and find the corresponding Nash intervals. We determine all players' thresholds that either break the player's behavior or keep them united. Finally, we study the dynamics in player's behavior around their thresholds mainly when small perturbations in the parameters occurs.

The combined effect of dispersal, asymmetry, and local dynamics on the asymptotic total population size

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Abstract

Many populations occupy spatially fragmented landscapes. How dispersal affects the asymptotic total population size is a key question for conservation management and the design of ecological corridors. The latest research provides a complete theoretical description of the asymptotic total population size response to dispersal in continuous time [1-2]. For discrete-time models, however, there are no similar results yet. We will analyze and describe all possible response scenarios for a Beverton-Holt discrete-time two-patch model and compare them with the analog logistic continuous-time model.

Dispersal is often asymmetric: wind, marine currents, rivers, or human activities produce a preferential direction of dispersal between connected patches. For the considered model, we will show that variable asymmetry allows populations to be led from a given response scenario to any other.

All previous numerical and theoretical findings determined that the possible response scenarios of the overall population size to increasing dispersal are monotonic or hump-shaped, which has become a common assumption in ecology. By considering different maps describing the local dynamics of the patches, we will show that the response of populations to variable dispersal can be very intricate.

[1] Arditi, R., Lobry, C., & Sari, T. (2015). Is dispersal always beneficial to carrying capacity? New insights from the multi-patch logistic equation, Theor. Popul. Biol., 106, 45–59.

[2] Gao, D., & Lou, Y. (2022). Total biomass of a single population in two-patch environments. Theor. Popul. Biol., 146, 1–14.

Exploring Chaos and Ergodic behavior of an Inductorless Circuit driven by Stochastic Parameters

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Abstract

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Complex Behaviors of a Predator-Prey Model Discretized by Piecewise Constant Arguments

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Abstract

It is well known that two kinds of equations, differential and difference equations, are used in the modelling of population dynamics. When the size of the population is rarely small or there are no overlapping generations, using difference equations is more appropriate. Moreover, it is also known that difference equations have richer dynamical properties than continuous systems. In view of this observation, in this talk we consider a predator-prey model, and first of all, we discretize the model with the help of piecewise constant arguments. The best advantage of this technique is that it avoids the existence of negative solutions in the discretized system. This method creates a bridge between differential and difference systems. Then we start the investigation of the types of fixed points and analyze the flip and Neimark-Sacker bifurcations using bifurcation theory and center manifold theorems. We also aim to construct the conditions that make our system chaotic. We support the results obtained here with some numerical experiments.

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Increasing delay as a strategy to prove stability in population models

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Abstract

By successively expanding scalar discrete-time systems, we increase their time delay and then establish a correlation between the stability of the new system and that of the original system. This approach is referred to as the expansion strategy, and it challenges the prevailing notion that an increase in delay results in a detrimental stability impact. Integrating the expansion strategy with the embedding technique leads to obtaining local and global stability results that enhance the existing knowledge on Schur stability. We show that this approach effectively proves global stability results for some population models.

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Stability results for nonlinear fractional difference equations

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Abstract

The asymptotic stability of nonlinear fractional difference equations with a Hilfer-like nabla operator is examined in this presentation. The results are new and advanced as a Hilfer-type nabla fractional difference that contains Riemann-Liouville and Caputo nabla difference as a particular case. To obtain the main results, linear scalar fractional difference equality, discrete comparison principle, and the basics of difference equations are utilized. In this talk, we present a Lyapunov second direct method for nonlinear discrete fractional systems. Asymptotic stability results are provided for some examples.

Approximate controllability of semilinear discrete control systems

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Abstract

The objective of this talk is to present some sufficient conditions for approximate controllability of semilinear control discrete systems as below:

$$\begin{aligned} x(n+1) &= A(n)x(n) + B(n)u(n) + h(n, x(n), u(n)), \quad n \in \mathbb{N}^* \\ x(0) &= x_0 \end{aligned}$$

where $\mathbb{N}^* = \mathbb{N} \cup \{0\}$, state variable $x(n) \in X$, control variable $u(n) \in U$, X and U are Hilbert spaces. $A \in l^\infty(\mathbb{N}, L(X))$, $B \in l^\infty(\mathbb{N}, L(U, X))$, $u \in l^2(\mathbb{N}, U)$, $L(U, X)$ represents the space of all bounded linear operator from U to X and $L(X, X) = L(X)$.

The results are obtained by the theory of difference equations, discrete Gronwall's inequality and suitable condition on the nonlinearity $h(n, x(n), u(n))$ as below.

The nonlinear term $h : \mathbb{N}^* \times X \times U \to X$ is a continuous Lipschitz function for Lipschitz constant $l_h > 0$. That is to say, for all $x_1,\ x_2 \in X$ and $u_1\ u_2 \in U$ we have that

$$||h(n, x_2, u_2) - h(n, x_1, u_1)|| \leq l_h\{||x_2 - x_1|| + ||u_2 - u_1||\}.$$

A new global exponential stability criterion for delay difference equations with applications to neural networks with delay

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Abstract

The objective of this talk is to present a new global exponential stability criterion for a general multidimensional delay difference equation, whose proof relies on a new induction argument. We emphasize that our result is applicable to nonautonomous equations presenting delay in the linear part. When the difference equation is periodic, we prove the existence of a periodic solution by constructing a type of Poincare map. The results are used to obtain stability criteria for a general discrete-time neural network model with delay in the leakage term and, as particular cases, we obtain new stability criteria for neural network models recently studied in the literature, in particular for low-order and high-order Hopfield and Bidirectional Associative Memory models. This presentation is based on the paper [1].

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Hyperchaos and bifurcations in models of coupled gas bubbles oscillations

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Abstract

In this talk we study the nonlinear dynamics in the models of oscillations of encapsu- lated micrometer size gas bubbles in a liquid. Such bubbles are currently used as ultrasound contrast agents and for targeted drug delivery and various types of oscillations can be either beneficial or undesirable, depending on application. Therefore, understanding of possible types of bubbles dynamics and their bifurcations under the variation of the control parame- ters is an important problem from an applied point of view. Moreover, the models of bubbles oscillations can be a

source of new universal bifurcation scenarios, as we demonstrate in this talk, and, hence, their investigation is interesting from a theoretical point of view. We consider mathematical models of two and three interacting gas bubbles and construct two dimensional chars of dynamical regimes in the control parameters space. We demonstrate that in both case there are hyperchaotic oscillations and propose several bifurcation scenarios for their onset. We also study appearance of chaotic and hyperhaotic attractors with addi- tional zero Lyapunov exponent in the case of three interacting bubbles. Finally, we discuss several aspect of synchronization of bubbles oscillations and existence of the synchronization and partial synchronization manifolds.

Zabczyk-Type Criteria for Stability of Discrete Dynamical Systems and Applications

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Abstract

We present nonuniform criteria of Zabczyk type for the detection of the nonuniform and uniform exponential stability of discrete variational systems. Starting from an ergodic theory approach introduced in [1] and from certain trajectory methods initiated and developed in [3-5], we present a new method of studying the stability of discrete dynamical systems. Our new results are based on some (nonuniform) convergence properties of series of nonlinear trajectories and the conditions take place only locally, on a subset of positive measure of the parameter space. As applications, we expose several robustness results for nonuniform/uniform exponential stability under additive and multiplicative perturbations as well as new Rolewicz type criteria, of nonuniform nature, for stability of skew-product semiflows. The presentation will be mainly based on the stability results in [2].

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Bessel functions on time scales and applications to partial dynamic equations

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Abstract

Following the ideas from [1] and [2], we introduce Bessel functions and modified Bessel functions of integer order on general time scales, and establish their basic properties. The main goal is to obtain fundamental solutions of certain partial dynamic equations. Even in the purely discrete case, we obtain new formulas for the fundamental solutions of the discrete wave equation and discrete diffusion equation with backward time differences.

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Environmental Policy and the Dynamics of Industrial Location and Residential Choice

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Abstract

Ecological challenges have a truly international dimension. In many instances, the causes of environmental issues are internationally interlinked and the consequences are not regionally confined. However, even when the causes and effects of environmental problems seem locally bounded, the flow of goods across borders and

international factor migration could transmit effects of these environmental problems and of the policies, which are implemented to deal with them, throughout the international economic network. A popular hypothesis is that individuals, especially those with higher human capital, migrate from polluted areas [1], whereas firms relocate from areas with strict environmental regulations because of higher operating costs. The latter hypothesis, known as the pollution haven hypothesis, suggests that firms tend to concentrate in areas with less stringent environmental regulations (see [4]). This leads to the phenomenon of exporting pollution and creates a markedly uneven distribution of polluting emissions ([5-6]). On closer inspection, however, the incentives for mobility are ambiguous: firms are attracted not only to low factor cost regions but also to regions with a larger local market; people are attracted not only to regions with less pollution but also to regions with better availability of goods. The aim of this paper is to show how environmental regulation may affect the long-run distribution of people and firms (and pollution), taking explicitly into account that their location decisions have different motivations. We find that only highly uneven environmental regulations lead to results corresponding to the pollution haven hypothesis: people concentrate in the region with high environmental standards and thus good environmental quality, whereas firms agglomerate in the region with low environmental standards and thus low environmental quality. In contrast, moderate environmental regulations in one region contributes to a better environmental quality in both regions.

We assume that the mobility choices of households and firms (the latter following human capital allocation decisions) are determined by an adaptive mechanism resembling the replicator dynamics. In particular, residential choices are driven by the utility differential $\Omega(\lambda,\eta) = \frac{u_1-\overline{u}}{\overline{u}}$ as follows

$$f(\lambda,\eta) = \lambda\left(1+\gamma_\lambda\Omega(\lambda,\eta)\right),$$

where $\overline{u} = \lambda u_1 + (1-\lambda)u_2$ and γ_λ is the adjustment speed; and human capital allocation depends on the remuneration differential $\Psi(\lambda,\eta) = \frac{w_1-\overline{w}}{\overline{w}}$ as follows:

$$g(\lambda,\eta) = \eta\left(1+\gamma_\eta\Psi(\lambda,\eta)\right),$$

where γ_η is the adjustment speed. After introducing the usual constraints on shares, the 2-D map corresponds to

$$T : (\lambda,\eta) \to (F(\lambda,\eta), G(\lambda,\eta))$$

where

$$\begin{aligned} F(\lambda,\eta) &= \left\{ \begin{array}{ccc} 0 & if & f(\lambda,\eta)\le 0 \\ f(\lambda,\eta) & if & 0<f(\lambda,\eta)<1 \\ 1 & if & f(\lambda,\eta)\ge 1 \end{array}\right. \\ G(\lambda,\eta) &= \left\{ \begin{array}{ccc} 0 & if & g(\lambda,\eta)\le 0 \\ g(\lambda,\eta) & if & 0<g(\lambda,\eta)<1 \\ 1 & if & g(\lambda,\eta)\ge 1 \end{array}\right. \end{aligned}$$

The 2-D map T has a piecewise smooth definition with flat branches. Its domain corresponds to the unit square $u = [0,1]\times[0,1]$.

The aim of the paper is to characterize the dynamics of the model and to study the emergence of multiple attracting equilibria and the evolution of their basins of attraction.

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Multi-dimensional chaos in the systems with pulsed action

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Abstract

Control of dynamical systems is one of the fundamental problems [1,5]. It attracts much attention of researchers due to both its theoretical background and great practical significance. The classical problems in this direction include chaos control, which corresponds to the suppression or stabilization of irregular oscillations. Chaos control methods have become widespread: stabilization of unstable periodic orbits of a dynamical system embedded in a chaotic attractor, and stabilization of unstable equilibrium states (fixed points). One of the most popular methods for solving such problems is the Pyragas method which is a method of auto-synchronization with delayed feedback [3].

In some situations, an external force applied on a system with an unstable regime can stabilize it, moreover it can initiate a system of periodic and quasi-periodic oscillations. A certain phase flow is observed in the system up to the saddle-node

bifurcation threshold, running away to infinity, and the addition of a periodic external action to such a system creates the possibility of stabilizing the regime [2,4]. The stabilization gives rise the synchronous response of the system, as well as quasi-periodic oscillations, with invariant curves of which doubling bifurcations can occur. We investigate the features of quasi-periodic oscillations in such a system, their destruction and the appearance of various chaotic attractors.

In the frame of work we consider in detail scenarios of multi-dimensional chaos formation that are observed when the trajectory is stabilized. We also discuss the universality of the observed phenomenon using the example of a different choice of the direction of external action, and thus, we describe the situations when such stabilization will not be observed. We consider situations when equilibrium states, or even a limit cycle, have already appeared in the autonomous subsystem and discuss the universality of the observed effects, in the context of the basic dynamical regime of the autonomous system. As the main objects we use the Rössler system on the threshold of saddle-node bifurcation and model of Anishchenko-Astakhov generator in the regime of divergency with trivial saddle-focal equilibrium point.

The work was prepared within the framework of the Basic Research Program, Project "Mirror Laboratories" at HSE University and partially supported by the Russian Science Foundation (project No. 20-71-10048, https://rscf.ru/project/20-71-10048/).

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Going in circles: From numerical solutions of an impulsive equation to delay equations on manifolds

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Abstract

Consider the stable delay equation

$$x'(t) = -ax(t) + bx(t - \tau),$$

where the constant parameters satisfy $a > |b|, \tau > 0$, with the impulsive condition $x(s^-) = c \Longrightarrow x(s^+) = c + d$, for $c, d > 0$, arbitrary positive constants. It is known that under certain additional conditions on the parameters, all solutions (with initial conditions $> c$) tend to a unique periodic solution of period greater than the delay.

Numerical evidence has motivated Hartung to conjecture that all solutions should tend to a unique periodic solution, under no further assumptions on the parameters. It was also recently proven that, even for such impulsive equations, numerical solutions tend to true solutions, lending credence to this numerical evidence and Hartung's conjecture.

We study an equivalent formulation of this conjecture from the point of view of delay equations on manifolds (RFDEs), the specific manifold in question being the circle, S^1. We describe two technical obstructions to proving this conjecture using the traditional theory of RFDEs.

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Derivation and analysis of discrete population models with delayed growth

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Abstract

Discrete delay population models are often considered as a compromise between single-species models and more advanced age-structured population models [1]. This talk is based on a recent work [2], where we provide a procedure for deriving discrete population models for the size of the adult population at the beginning of each breeding cycle and assume only adult individuals reproduce. This derivation technique includes delay to account for the number of breeding cycles a newborn individual remains immature and does not contribute to reproduction. These models include a survival probability (during the delay period) for the immature individuals, since these individuals have to survive to reach maturity and become members of what we consider the adult population. We discuss properties of this class of discrete delay population models and show that there is a critical delay threshold. The population goes extinct if the delay exceeds this threshold. We apply this derivation procedure to two well-known population models, the Beverton–Holt and the Ricker population model.

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Wold-type decompositions for m-isometries on Hilbert spaces

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Abstract

We present classes of m-isometric operators on Hilbert spaces, defined for a positive integer m, which admit Wold-type decompositions in the sense of S. Shimorin. Among these operators we identify some sub-Brownian m-isometries and their m-Brownian unitary extensions. We describe also the sub-Brownian 3-isometries T in

terms of their Cauchy dual operator and we show, that under certain kernel conditions, such an operator T and its restriction to the range simultaneously admit Wold-type decompositions.

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Isotropic stationary solutions of lattice differential equations

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Abstract

Lattice differential equations can have an infinite number of stationary solutions. The solutions are in general described as roots of a countable system of algebraic equations. We focus on isotropic stationary solutions — solutions which: 1) can be governed by a finite number of algebraic equations, 2) generalize the notion of periodic solutions. In this talk we aim to present elementary properties of the isotropic solutions and, utilizing the results on perfect colorings (graph colorings in which the colors of k-neighbors of a vertex depend solely on the color of the central vertex), show that there can be uncountably many distinct — with respect to various lattice symmetries — stationary solutions on rectangular, hexagonal, and triangular grid which can be described by a pair of algebraic equations.

Properties of a two-dimensional non-invertible rational map

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Abstract

In this talk we will explore additional properties of a two-dimensional map denoted by $T : (x', y') = \left(-\frac{ax}{1+y^2}, x + by\right)$, which depend on the parameters a and b. While this map has been examined in numerous articles $[1-3]$, many aspects of its dynamical analysis remain unexplored. Specifically, we delve into the invertibility of the map and demonstrate that it conforms to a $Z_1 - Z_2$ type through critical curve computations. Notably, the map exhibits simplicity through symmetry, allowing for explicit expressions of both stable and saddle period-two orbits. Furthermore, we observe various codimension-one bifurcations including saddle-node, period-doubling, and Neimark-Sacker bifurcations. Additionally, we identify a subcritical pitchfork bifurcation.

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Numerical analysis of the car following problem with different variable-order differential operators

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Abstract

The main task of the car following model (CFM) is to control the behavior of the vehicle, taking into account the movement of the preceding vehicle in the same lane. It can be said that the basic values that determine the state of a given car - namely, its position and speed - are manipulated so that its state is as close as possible to that of the leading car [6,16,19]. Research on the CFM problem was initiated in 1935 in [7]. Various approaches can be distinguished that were intended to solve traffic flow problems. One in which the behavior of the cars can be compared to a train, in which the lead vehicle controls the movement of the vehicles (which are like "cars") like a locomotive on a train, is known as a platoon [14,20,21]. Another idea are adaptive cruise control (ACC). These allow vehicles to solve traffic flow problems on their own, and the latest of these, which include a two-way control model (BCM) [9,10,12] keep the car as far away from the driving car as the car behind it. Due to the instability of the classical CFM model, in which a small perturbation in the equilibrium state causes congestion and collisions, various modifications of the aforementioned approaches are known. In our work, we propose a modification of the CFM model using fractional operators. The application of fractional operators to car following models introduces a new perspective to the analysis of vehicle dynamics. For the analysis, we used Grünwald-Letnikov [8,3] and Caputo [1] operators. Theoretical considerations were illustrated by numerical simulations.

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A detailed study of the behavior of the solutions of a general second-order system of difference equations

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Abstract

Motivated and inspired by the references [1]-[6], our aim in this talk is to establish results on global stability, existence of periodic and oscillatory solutions of a general second-order system of difference equations defined by two continuous and homogeneous functions on $(0, +\infty)^2$ one is of degree zero and the other is of degree $s \in \mathbb{R}$. The obtained results are showed on some concrete systems in details.

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Robust Invariant Sets for Systems Affected by State-Dependent Disturbances

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Abstract

Invariant sets have received much interest in constrained and robust control [2,8] and set-valued analysis of dynamical systems [1]. The theoretical and numerical properties of invariant sets of autonomous contractive discrete-time linear systems subject to a bounded additive disturbance is well-understood [4, 6, 7, 10, 11]. In particular it is known that for a stable system subject to a bounded additive disturbance that the intersection of all invariant sets is non-empty i.e. there is a smallest invariant set – the minimal Robust Postively Invaraint (mRPI) set – contained in all other invariant sets.

Recent work has extended the analysis of linear systems subject to bounded state-independent disturbances. For example to systems subject to probabilistic disturbances [5], nonlinear systems subject to disturbances with state-dependent bounds [5,9], constrained systems [3, 13] and the reachability of invariant sets in [12].

We consider the invariant sets of linear systems subject to state-dependent disturbances. We develop a lift-and-project set-valued map which equivalently describes the dynamics of the system in a state-disturbance space and relate the invariant sets – in particular mRPI set – of the system to the fixed points of this set-valued map.

This enables the mRPI set to be constructed via set iterates by recursively applying set-valued maps on the equivalent extended dynamic representation of the state-disturbance system, we characterise the Robust Positively Invariant (RPI) sets of stable systems subject to state-dependent locally bounded disturbances.

For these systems, the mRPI set does not necessarily exist. However, when it does, we prove that it is found in the family of fixed points of the set-valued maps. This allows the mRPI to be characterised by a much smaller collection of sets compared to the wider collection of all closed RPI sets.

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Behaviour of Solutions of a class of 2nd Order 2D Neutral Delay Difference Systems

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Abstract

The present work deals with the study of the dynamic behaviour of all vector solutions of a second order neutral delay difference system with constant coefficients of the form:

$$\Delta^2 \begin{bmatrix} \phi(m) - q\phi(m-l) \\ \psi(m) - q\psi(m-l) \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \begin{bmatrix} \phi(m-\tau) \\ \psi(m-\gamma) \end{bmatrix}$$

for $m \geq m_0 > \rho = \max\{l, \tau, \gamma\}$ by constructing the characteristic equations, where $q \in \mathbb{R} - \{0\}$, $b_1, b_2, b_3, b_4 \in \mathbb{R}$, $l > 1, \tau, \gamma \in \mathbb{Z}^+$. Some examples are cited to verify our results.

Stability and evolutionary stability of certain non-autonomous competitive systems of difference equations

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Abstract

My presentation would discuss the dynamics of non-autonomous competitive systems of difference equations with asymptotically constant coefficients. We are mainly interested in global attractivity results for such systems and the application of such results to evolutionary population competitive models. We develop a method for proving these types of systems. Finally, a synopsis on evolutionary game theory and how some of these systems can be applied to evolutionary dynamics will be discussed. This was joint paper with M. Kulenović, M. Nurkanović, and Z. Nurkanović.

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Fuzzy Calculus on Time Scales

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Abstract

In this talk, we introduce a novel concept of derivative and integral called the granular delta derivative and integral for fuzzy functions on time scales, based on a granular approach. Our objective is to provide an advanced tool for the analysis of fuzzy dynamic equations on time scales. The key contribution of this study is that we establish the concept of the limit of fuzzy functions on time scales, employing a granular metric on the fuzzy number set. This limit allows us to provide an equivalent definition of the derivative of fuzzy functions via limit-based language. Then, we introduce the granular delta derivative and integral for fuzzy functions on time scales based on the granular distance and horizontal membership functions. Several vital characteristics of these concepts are rigorously demonstrated. This talk is the introduction to the second one titled “Applications to Fuzzy Dynamic Equations on Time Scales”

A Discrete SEIRS model for the COVID-19

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Abstract

The COVID-19 pandemic has led to a dramatic loss of human life worldwide, economic and social disruption. Demirci in [1] was proposed a novel mathematical model to understand how to spreed COVID-19 pandemic within population. In this study, we consider a special discrete case of Demirci's continious SEIRS model. The model consists of five compartments: $S_n, E_n, I_{1n}, I_{2n} and R_n$. S_n is composed of the susceptible individuals in the population. The infectious group is divided into three classes that are exposed E_n, infective who are diagnosed with COVID-19 I_{1n} and infective who are not diagnosed (both symptomatic and asymptomatic) I_{2n}. The individuals in the exposed group are infected but due to the incubation period of the virus they are not contagious until the last two days of this period. The diagnosis rate of these individuals is also quite low even though they are tested. In this study the equilibrium points of discrete-time COVID19 model will be introduced and proposed their stability analysis. Also the basic reproduction number (R_0) will be obtained with a suitable method.

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Controlling a pest species: Multiple control mechanisms in a host-parasitoid system

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Abstract

We develop discrete-time host-parasitoid models to examine the impact of multiple control mechanisms to manage a pest species. The models include developmental stage-structure in both species and continuous or periodic (via an impulsive system) application of pest control through pesticide spraying and parasitoid supplementation. We analytically study the dynamical outcomes of these models, including the global asymptotic stability of the boundary equilibria, global attractivity of a pest-eradication cycle, and system persistence. We also examine how varying the control measures and system properties may lead to different pest control outcomes. We observe that the effectiveness of integrated pest management strategies may be dependent on several factors including the timing of events, the type of attractor, and the initial conditions. Moreover, in certain scenarios the combined control strategies may produce worse outcomes than a single control strategy.

Exploring chaos of near-identity maps using interpolating vector fields

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Abstract

We shall present interpolating vector fields (IVFs) as a theoretical and numerical tool to study dynamics of near-identity maps of any dimension. After discusing its theoretical properties, we will consider some numerical applications. We will use IVFs to illustrate the chaotic dynamics near double resonances of quasi-integrable 4D symplectic maps. Finally, some applications of IVFs to the study of dissipative systems will be also discused.

Global dynamics of a mosquito-borne disease model in heterogeneous environments

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Abstract

In this talk, I will introduce a reaction-diffusion mosquito-borne disease model with spatial heterogeneity and general incidence rates. The basic reproduction ratio R0 for this model is introduced and the threshold dynamics in terms of R0 are obtained. In the case where the model is spatially homogeneous, the global asymptotic stability of the endemic equilibrium is proved when $R_0 > 1$. Under appropriate conditions, we investigate the asymptotic profiles and monotonicity of R0 with respect to the heterogeneous diffusion coefficients. Numerically, the proposed model is applied to study the dengue fever transmission. Via performing simulations on the impacts of certain factors on R0 and disease dynamics, we find some novel and interesting phenomena which can provide valuable information for the targeted implementation of disease control measures.

Difference Equation Models of Population Growth involving Distributed Delay

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Abstract

We introduce a class of single species difference equation models with distributed delay in the reproductive process and a survival function that accounts for survival pressure during that delay period that generaizes the model discussed in [1] that only considered discrete delay. The resulting delay recurrences are aimed at modeling the mature population for species in which individuals reach maturity after at least τ and at most $\tau + \tau_M$ breeding cycles. This differs from existing discrete population models with distributed delay that consider delay in the fitness (per-capita) growth rate. For the general set-up, but under realistic model assumptions, we prove the existence of a critical delay threshold, $\widetilde{\tau}_c$. For given model parameters and given delay kernel length τ_M, if each individual takes at least $\widetilde{\tau}_c = \widetilde{\tau}_c(\tau_M)$ to reach maturity, then the population is predicted to go extinct, a reasonable biological consequence. We show that the positive equilibrium is decreasing in both τ and τ_M. In the case of a constant reproductive rate, we provide an equation to determine $\widetilde{\tau}_c$ for fixed τ_M, and similarly, provide a lower bound on the kernel length, $\widetilde{\tau}_M$ for fixed τ such that the population goes extinct if $\tau_M \geq \widetilde{\tau}_M$. We compare $\widetilde{\tau}_c$ and $\widetilde{\tau}_M$ for the model with different distributions, e.g., with the Dirac-delta, binomial, uniform, linear increasing, and linear decreasing kernels and show that if all else is the same, the

Dirac-delta distribution has the highest critical delay threshold of any distribution and that to avoid extinction it is best if all individuals in the population have the shortest delay possible. We apply the model derivation to a Beverton–Holt model and discuss the global dynamics.

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Demonstration of a Standalone GUI Available on Github for Analyzing Planar Difference Equations

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Abstract

It is first shown why phase-plane analysis of planar maps has not been as useful as it has been for planar systems of ordinary differential equations. To rectify this issue the next-iterate operator associated with the nullclines and their associated root-curves was introduced in [1]. This provides an elementary approach that can be applied to analyze discrete planar models. A standalone GUI to obtain augmented phase portraits was created and will be demonstrated and applied to several planar systems. The GUI is available at

`https://github.com/sabrinaheike/AugmentedPhasePortrait`

for Windows, Mac, and Linux operating systems. Although the GUI was created iusing Matlab, it does not require the user to have Matlab on thier system. They do require Matlab runtime, a progam that is free and downloadable from

`https://www.mathworks.com/products/compiler/matlab-runtime.html`

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Periodic solutions to differential equations with distributed delay

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Abstract

The distributed delay differential equation, as a mathematical model, can reasonably reflect the time delay effect in the development process of things, and is widely used in many fields such as infectious disease prevention and control, population growth, and industrial production. This report mainly introduces some results of me and my collaborators applying critical point theory to study periodic solutions of distributed delay differential equations, including the existence, multiplicity, and estimation of the number of periodic solutions for generalized Kaplan-Yorke type equations, Kennedy equations, and other equations.

Spectral theory of linear Hamiltonian differential systems and discrete symplectic systems: Unified

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Abstract

In this talk, we present an introduction to the spectral theory of linear relations associated with dynamic symplectic systems on time scales. In particular, we focus on the minimal and maximal linear relations and their connection. This development enables us to unify and extend analogous results from the theory of linear Hamiltonian differential and difference systems and discrete symplectic systems, such as [1-2]. It is worth noticing that our theory is built in the more general setting of linear relations in contrast to the traditional approach through linear operators (especially for the second-order differential equations), which is almost impossible in this case.

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Approximate Boundaries of Arnold Tongue

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Abstract

An Arnold tongue of a planar map is a region of parameters in which periodic orbits appear on the invariant circle arising from the Hopf (Neimark-Sacker) bifurcation. Although conditions were given for the existence of those tongues, it is still difficult to compute their boundaries of them. We give a method to compute approximately all common terms and the first different term for the boundaries. Then, discussing higher order derivatives and Hölder smoothness for implicit functions, we use the inducing relation of parameters to compute approximately boundaries of tongues for given planar maps and apply the method to a planar discrete time prey–predator model.

Takens Theorem for a partially hyperbolic dynamics

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Abstract

Takens Theorem for a partially hyperbolic dynamics provides a normal linearization along the center manifold. In this talk, we introduce a nonautonomous version of Takens Theorem under non-resonance conditions formulated in terms of the dichotomy spectrum.

Periodic solutions and stability of a discrete mosquito population model with periodic parameters

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Abstract

In this talk, we introduce our recent study on a discrete mosquito population model with periodic parameters that takes into account the seasonal variation of environments. We focus on finding a positive periodic solution that is asymptotically stable and attracts all positive solutions. The existence, uniqueness, and stability of periodic solutions of the model are investigated. The two most attractive findings are: (1) the instability of the origin implies that the model has a unique asymptotically stable positive periodic solution and attracts all positive solutions; (2) the local stability of the origin implies its global asymptotic stability. We also give a necessary and sufficient condition for the origin to be stable. The talk will end with some numerical examples to illustrate our theoretical results. This is a joint work with Xiaoping Wang, Yu Gu, Jinhua Wang, and Fangfang Liao at Xiangnan University. We are deeply indebted to Professor Jianshe Yu at Guangzhou University for his insightful and stimulating suggestions throughout the whole writing.

Elementary methods to prove and improve the Cushing-Henson conjecture

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Abstract

In this talk, we will introduce rather elementary methods to prove and improve the Cushing-Henson conjecture proposed in 2002. The condition for the existence, uniqueness, and positiveness of the periodic solutions has been relaxed. This is a joint work with Professor Bo Zheng and Professor Jianshe Yu at Guangzhou University.

Dichotomy and Asymptotic Equivalence for Systems in Banach Spaces

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Abstract

The bounded solutions of two systems are said to be asymptotically equivalent if the sets of bounded solutions are homeomorphic and the distance of any corresponding two bounded solutions tends to zero as time goes to infinite. In this talk, we will introduce our work on the dichotomy and asymptotic equivalence of bounded solutions for difference equations in Banach spaces joint with Yujie Bai and Haiyang Huang.

References:

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Positive solutions of discrete Dirichlet problems involving the mean curvature operator

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Abstract

In this talk, we will introduce some results on the positive solutions for some nonlinear discrete Dirichlet boundary value problems involving the mean curvature operator by using critical point theory. First, some sufficient conditions on the existence of infinitely many positive solutions are given. We show that, the suitable oscillating behavior of the nonlinear term near at the origin and at infinity will lead to the existence of a sequence of distinct nontrivial positive solutions. Then, the existence of at least two positive solutions is established when the nonlinear term is not oscillatory both at the origin and at infinity, we show that this result is sharp on some sense. Examples are also given to illustrate our main results at last.

Stability and periodicity in a mosquito population suppression model composed of two sub-models

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Abstract

In this talk, we propose a mosquito population suppression model which is composed of two sub-models switching each other. We assume that the releases of sterile mosquitoes are periodic and impulsive, only sexually active sterile mosquitoes play a role in the mosquito population suppression process, and the survival probability is density-dependent. For the release waiting period T and the release amount c, we find three thresholds denoted by $T^\star$, $g^\star$, and $c^\star$ with $c^\star > g^\star$. We show that the origin is a globally or locally asymptotically stable equilibrium when $c \geq c^\star$ and $T \leq T^\star$, or $c \in (g^\star, c^\star)$ and $T < T^\star$. We prove that the model generates a unique globally asymptotically stable T-periodic solution when either $c \in (g^\star, c^\star)$ and $T = T^\star$, or $c > g^\star$ and $T > T^\star$. Two numerical examples are provided to illustrate our theoretical results.

Merging of a Chaotic Attractor in a Discontinuous Map Caused by a Pitchfork Bifurcation

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Abstract

Mechanisms that induce the appearance, disappearance, as well as various transformations of chaotic attractors are of great interest in many applications of nonlinear dynamics. The latter class of phenomena, also known as crises, has been extensively studied both numerically and theoretically in one-dimensional and multidimensional dynamical systems. It has been shown that these phenomena are associated with homoclinic bifurcations of unstable periodic orbits. In particular, it is known that a bifurcation in which some bands of a multi-band chaotic attractor merge pairwise – a so-called merging bifurcation – is in general associated with a homoclinic bifurcation of an unstable cycle with a negative eigenvalue (multiplier).

In the present work, we investigate the dynamics of a 1D map which acts as a model of an H-bridge inverter with hysteresis control. Due to the applied control strategy, the map may have several discontinuities inside the absorbing interval. We demonstrate that as a parameter is varied, this map exhibits an unusual type of a merging bifurcation. Indeed, the cycle involved in this bifurcation loses its stability through a supercritical pitchfork bifurcation and consequently has a positive multiplier. We describe the mechanism causing this bifurcation and discuss how our findings fit the existing theory of bifurcations of chaotic attractors.

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